

Four-fermion operators in Heavy Meson χ Perturbation Theory*

C.-J. David Lin

NCTU & NCTS

PPP7, NTU

09/06/07

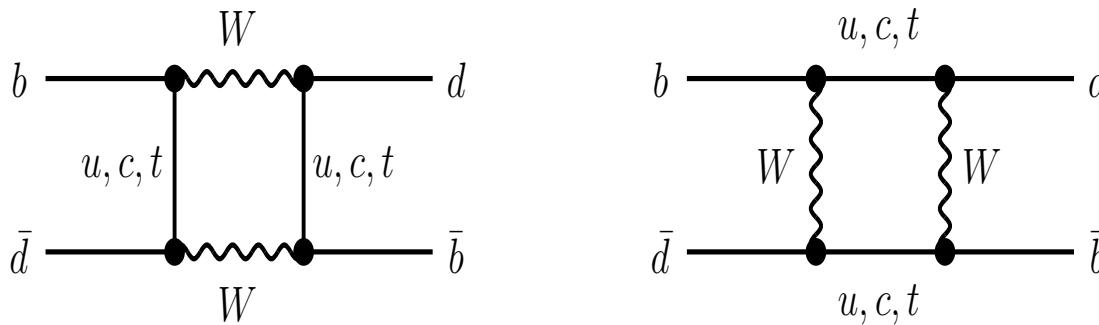
*W.Detmold and C.-J.D.L., hep-lat/0612028, to appear in PRD.

Outline

- Introduction
- Heavy meson χ PT (HM χ PT)
- Four-fermion operators
- Conclusions and outlook

$B^0-\bar{B}^0$ mixing

- The Standard Model contribution:



$$\mathcal{O}_1 = \bar{b}_\alpha \gamma^\mu (1 - \gamma_5) d_\alpha \bar{b}_\beta \gamma^\mu (1 - \gamma_5) d_\beta.$$

- BSM

$$\mathcal{O}_2 = \bar{b}_\alpha (1 - \gamma_5) d_\alpha \bar{b}_\beta (1 - \gamma_5) d_\beta, \quad \mathcal{O}_3 = \bar{b}_\alpha (1 - \gamma_5) d_\beta \bar{b}_\beta (1 - \gamma_5) d_\alpha.$$

$$\mathcal{O}_4 = \bar{b}_\alpha \gamma^\mu (1 - \gamma_5) d_\alpha \bar{b}_\beta \gamma^\mu (1 + \gamma_5) d_\beta, \quad \mathcal{O}_5 = \bar{b}_\alpha \gamma^\mu (1 - \gamma_5) d_\beta \bar{b}_\beta \gamma^\mu (1 + \gamma_5) d_\alpha.$$

Lattice QCD on one slide

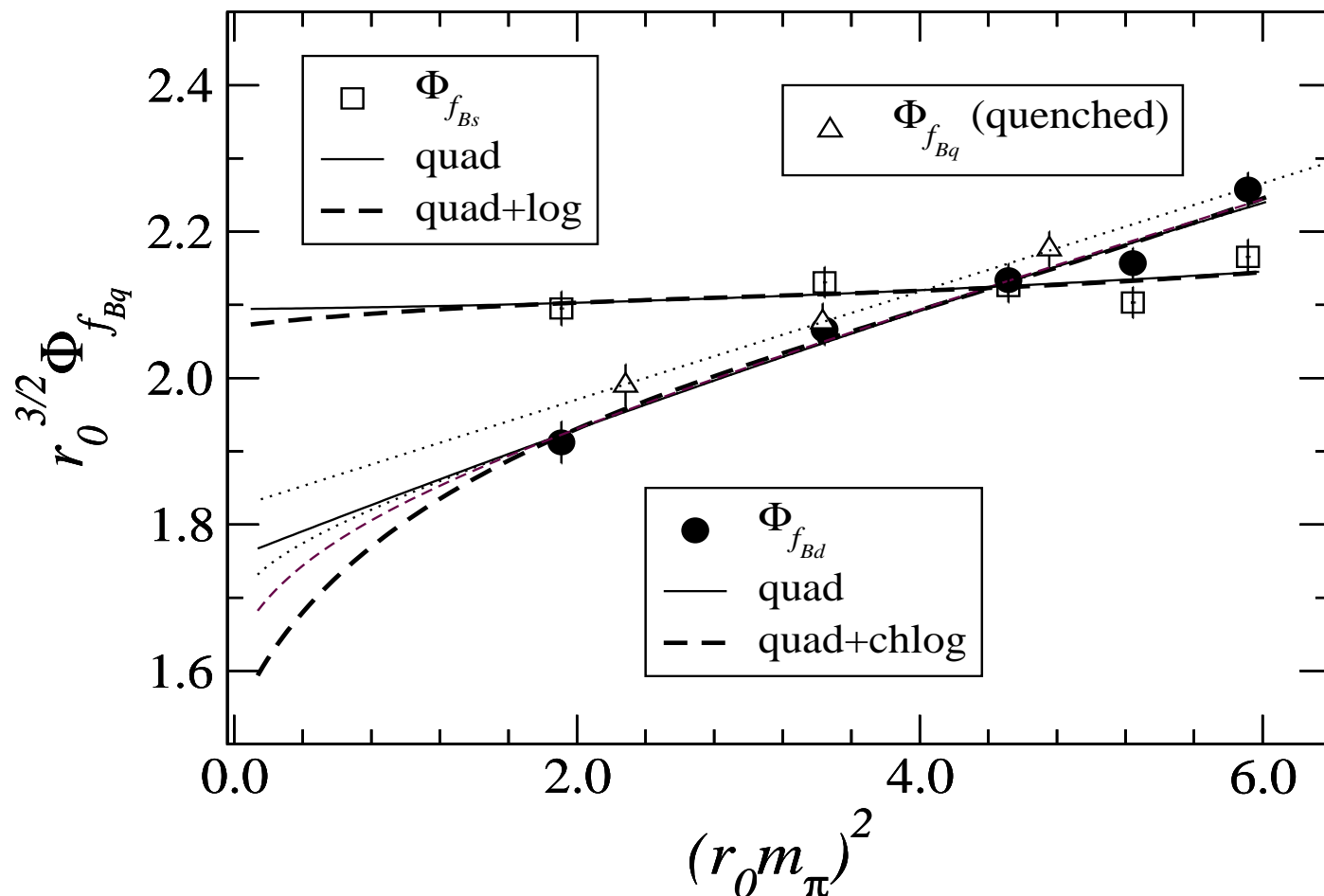
- Partial quenching (limited by algorithm):

$$\begin{aligned}
 \langle \mathcal{O} \rangle &= \frac{1}{Z} \int [dA] [d\psi] [d\bar{\psi}] \mathcal{O} e^{-S_{\text{QCD}}} \\
 &= \frac{1}{Z} \int [dA] \prod_i \det (\not{D} + m_i) e^{-S_g[A]} \mathcal{F} \left(\frac{1}{\not{D} + m_i} \right) \\
 &\xrightarrow{\text{PQ}} \frac{1}{Z_{\text{PQ}}} \int [dA] \prod_{i,gi,si} \left[\frac{\det (\not{D} + m_i)}{\det (\not{D} + m_{gi})} \right]_{m_{gi}=m_i} \det (\not{D} + m_{si}) e^{-S_g[A]} \mathcal{F} \left(\frac{1}{\not{D} + m_i} \right).
 \end{aligned}$$

- Finite lattice spacing: $1/a \sim 3 \text{ GeV} < m_b$.
- Finite volume: $L \sim 3 \text{ fm}$.
- Extrapolate to physical quark masses and volume using EFT.

Light-quark mass extrapolation

$$\Phi_{f_B} = \alpha_0 [1 + \text{loop}(L, m_\pi, \mu)] + \alpha_2(\mu) m_\pi^2.$$



EFT for LQCD

- Partially quenched χ PT, based on $SU(6|3)_L \otimes SU(6|3)_R \rightarrow SU(6|3)_V$.

$$\Sigma = e^{2i\phi/f} \quad , \quad \langle \Sigma \rangle = \mathbf{1} \quad , \quad \Sigma \xrightarrow{SU(6|3)_L \otimes SU(6|3)_R} L \Sigma R^\dagger,$$

where ϕ is a 9×9 matrix containing Goldstone meson fields.

- Same χ symmetry breaking pattern as “full QCD”.
- Combining HQ spin symmetry and χ symmetry \rightarrow **HM χ PT** (HL mesons).
 - Heavy mesons are almost on-shell $p_\mu = M_B v_\mu + k_\mu$.
 - The velocity superselection rule.
 - HQET-like procedure leading to $1/M_B$ expansion.
- An important issue is **how to couple HL mesons to Goldstone mesons**.
 \Rightarrow Normally assume $M_\pi \ll \Lambda_{\text{QCD}} \sim \Lambda_\chi \ll M_B$ and discard $\mathcal{O}(M_\pi/M_B)$.
- $\int \rightarrow \sum$ for Goldstone loops to obtain FV effects.

HM χ PT

Basic ingredients

- Heavy-quark spin (HQS) symmetry at $M_B \rightarrow \infty$
 - Covariant field (bosonisation of interpolating operators $\bar{q}\Gamma^\mu Q$)

$$H_a = \frac{1 + \not{v}}{2} [B_a^{*\mu} \gamma_\mu + iB_a \gamma_5],$$

$$H_a \xrightarrow{\text{HQS}} SH_a, \quad H_a \xrightarrow{SU(6|3)_L \otimes SU(6|3)_R} H_b U_{ba}^\dagger.$$

- Coupled to Goldstone fields in:

$$\xi = \sqrt{\Sigma}, \quad \xi \xrightarrow{SU(6|3)_L \otimes SU(6|3)_R} L\xi U^\dagger = U\xi R^\dagger.$$

- Combine H and ξ to cancel the effect of U .
- While L and R are global, U is local \Rightarrow need to be gauged.

HM χ PT

The HM Lagrangian up to $\mathcal{O}(1/M_B)$

- Invariant under chiral and HQS transformation

$$\mathcal{L}_{\text{HM}} = -i \text{Tr} \bar{H}_a v_\mu (\partial^\mu \delta_{ab} + i V_{ab}^\mu) H_b + g \text{Tr} \bar{H}_a H_b \gamma_\nu \gamma_5 A_{ba}^\nu + \underbrace{\frac{\lambda_2}{M_B} \text{Tr} \bar{H}_a \sigma_{\mu\nu} H_a \sigma^{\mu\nu}}_{\mathcal{O}(1/M_B)},$$

$$V_\mu, A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger).$$

- Propagators :

$$- B : \frac{i}{2(v \cdot k + i\epsilon)} \xrightarrow{x\text{-space, rest } B} \theta(t) \delta^{(3)}(\vec{x}).$$

$$- B^* : \frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v \cdot k - \Delta + i\epsilon)}.$$

- Unknown parameters (Low Energy Constants):

$$- g : B-B^*-(\pi, K, \eta) \text{ coupling.}$$

$$- \lambda_2 : \Delta = M_{B^*} - M_B = -8 \frac{\lambda_2}{M_B} \sim 50 \text{ MeV at physical } M_B.$$

Leading-order $\text{HM}\chi\text{PT}$ $\Delta B = 2$ operators

$$\begin{aligned}\mathcal{O}_{LL} &= \bar{b} \Gamma_{LL} d_L \bar{b} \Gamma_{LL} d_L, \\ \mathcal{O}_{LR} &= \bar{b} \Gamma_{LR}^{(1)} d_L \bar{b} \Gamma_{LR}^{(2)} d_R,\end{aligned}$$

invariant under the spurion transformation

$$\Gamma_{LL} \longrightarrow S \Gamma_{LL} L^\dagger, \quad \Gamma_{LR}^{(1)} \longrightarrow S \Gamma_{LR}^{(1)} L^\dagger, \quad \Gamma_{LR}^{(2)} \longrightarrow S \Gamma_{LR}^{(2)} R^\dagger .$$

The operators are

$$\mathcal{O}_1^{\text{HM}\chi\text{PT}} = \beta_1 [(\xi B^\dagger) (\xi \bar{B}) + (\xi B_\mu^{*\dagger}) (\xi \bar{B}^{*,\mu})] ,$$

$$\mathcal{O}_{2(3)}^{\text{HM}\chi\text{PT}} = \beta_{2(3)} (\xi B^\dagger) (\xi \bar{B}) + \beta'_{2(3)} (\xi B_\mu^{*\dagger}) (\xi \bar{B}^{*,\mu}) ,$$

$$\begin{aligned}\mathcal{O}_{4(5)}^{\text{HM}\chi\text{PT}} &= \beta_{4(5)} (\xi B^\dagger) (\xi^\dagger \bar{B}) + \hat{\beta}_{4(5)} (\xi^\dagger B^\dagger) (\xi \bar{B}) \\ &+ \beta'_{4(5)} (\xi B_\mu^{*\dagger}) (\xi^\dagger \bar{B}^{*,\mu}) + \hat{\beta}'_{4(5)} (\xi^\dagger B_\mu^{*\dagger}) (\xi \bar{B}^{*,\mu}) .\end{aligned}$$

HQ symmetry breaking from the operators

- The $\Delta B = 2$ operators:

$$\mathcal{O}_i^{\text{HQET}} = \tilde{b}\Gamma_1 d b^\dagger \Gamma_2 d + b^\dagger \Gamma_1 q_a \tilde{b} \Gamma_2 d.$$

- The HQ spin operator (flip HQ spin):

$$S_b^3 = \epsilon^{ij3} [b^\dagger \sigma_{ij} b - \tilde{b} \sigma_{ij} \tilde{b}^\dagger].$$

- Spin relations:

$$\{S_b^3, \mathcal{O}_1^{\text{HQET}}\} |B\rangle = \{S_b^3, \mathcal{O}_1^{\text{HQET}}\} |\bar{B}\rangle = 0,$$

$$\{S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}\} |B\rangle \neq 0, \quad \{S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}\} |\bar{B}\rangle \neq 0,$$

$$[S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}] |B\rangle \neq 0, \quad [S_b^3, \mathcal{O}_{2,3,4,5}^{\text{HQET}}] |\bar{B}\rangle \neq 0.$$

Chiral extrapolation for $B^0 - \bar{B}^0$ matrix elements

- One-loop diagrams



- Extrapolation formulae

$$\langle \bar{B}^0 | \mathcal{O}_1 | B^0 \rangle = \beta_1 [1 + \text{Log}(m_\pi, L, \mu)] + \gamma_1(\mu) m_\pi^2,$$

$$\langle \bar{B}^0 | \mathcal{O}_{2(3)} | B^0 \rangle = \beta_{2(3)} [1 + \text{Log}(m_\pi, L, \mu)] + \beta'_{2(3)} \text{Log}'(m_\pi, L, \mu) + \gamma_{2(3)}(\mu) m_\pi^2,$$

$$\langle \bar{B}^0 | \mathcal{O}_{4(5)} | B^0 \rangle = \beta_{4(5)} [1 + \text{Log}(m_\pi, L, \mu)] + \beta'_{4(5)} \text{Log}'(m_\pi, L, \mu) + \gamma_{4(5)}(\mu) m_\pi^2,$$

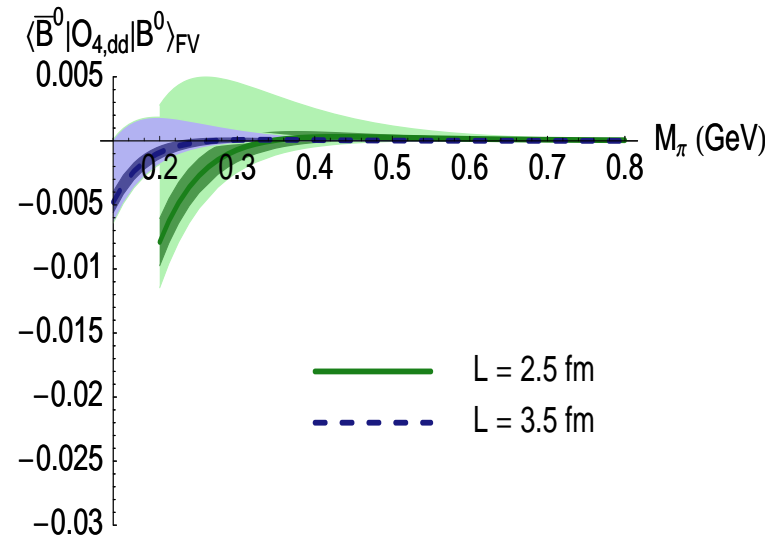
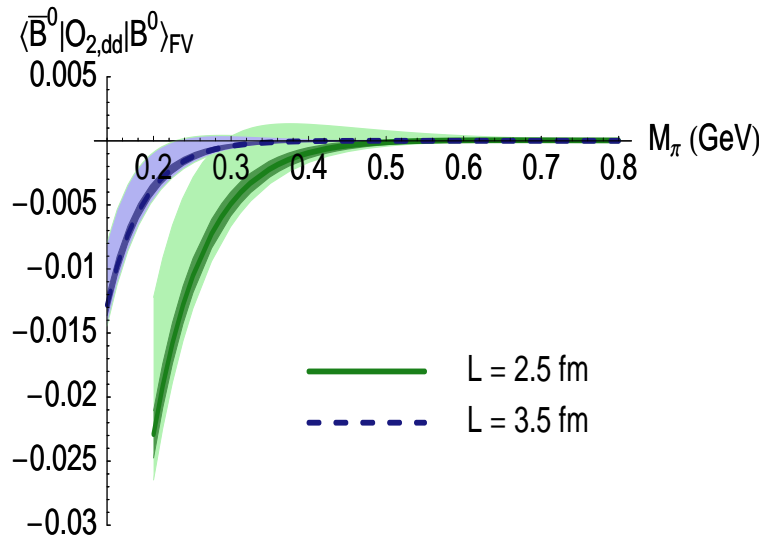
- Conventional wisdom is not applicable to the BSM operators.

Plots for FV effects

- FV effects in B_B are around 5%.

D.Arndt and C.-J.D.L, PRD70:014503 (2004).

- BSM matrix elements:



Conclusions and outlook

- Conventional wisdom in chiral extrapolation:

$$\beta(1 + \text{Log}) + \gamma m_\pi^2.$$

- Chiral extrapolation for BSM $B^0 - \bar{B}^0$ mixing matrix elements:

$$\beta(1 + \text{Log}) + \beta' \text{Log}' + \gamma m_\pi^2.$$

- The origin of the complication:

⇒ HQ symmetry breaking effects.

⇒ LO $B^* - \bar{B}^*$ matrix element is accompanied by a different LEC.

- On-going work:

⇒ $\Delta B = 0$ operators for spectator effects in B and Λ_b lifetimes.