June 8, 2007 at PPP7

Probing the octant of θ_{23} in neutrino factories: a combined analysis of appearance and disappearance modes at the magic baseline

G.-L. Lin

National Chiao-Tung U.

Taiwan

What we know about the neutrino oscillation parameters

Super-K I+II Preliminary

1.9×10^{-3} eV^2 $<|\Delta m^2_{31}|<$ 3.1×10^{-3} eV^2
$\sin^2 2\theta_{23} > 0.93$ at 90% CL
Best fit: $|\Delta m^2_{31}| = 2.5\times10^{-3}$ eV^2
$\sin^2 2\theta_{23} = 1$

K. Inoue at ICHEP06, Moscow

7.21×10^{-5} eV^2 < Δm^2_{12} < 8.63×10^{-5} eV^2

$0.267 < \sin^2\theta_{12} < 0.371$

Best Fit: 7.92×10^{-5} eV^2

$\sin^2\theta_{12}=0.314$

$\sin^2 2\theta_{13} < 0.124$

@2σ

G.L. Fogli, E. Lisi, A. Marrone and A. Palazzo
Progress in Particle and Nuclear Physics 2006

What we don't know about the neutrino mixing parameter

(a)The sign of Δm^2_{31}

LBL+ATM (magnetized iron calorimeter),

Very Long Baseline Experiment (matter effect)

(b)The octant of mixing angle θ_{23}

LBL+ATM,

Very Long Baseline Experiment (matter effect)

(c)The value of mixing angle θ_{13}

Reactors (Daya Bay, Double Chooz), T2K, NOνA

(d) The value of CP violation phase

For references on parameter degeneracy in LBL experiments and 3 flavor effects in atmospheric neutrinos, see T. Schwetz in ICHEP06, Moscow

For proposals of very long baseline experiments, see
I. Mocioiu and R. Shrock, Phys. Rev. D 62, 053017 (2000); V. D. Barger, S. Geer, R. Raja and K. Whisnant, Phys. Lett. B485, 379 (2000); V. D. Barger, S. Geer, R. Raja and K. Whisnant, Phys. Rev. D 62, 013004 (2000); F. DeJongh, arXiv:hep-ph/0203005

For relating matter effects in ν_μ survival probability to θ_{23}, see Antusch *et al.*, hep-ph/0404268; D. Choudhury and A. Datta, JHEP07, 058 (2005).

For experiments probing θ_{13} and CP violation phase,see
K. Inoue and R. Rameika at ICHEP06, Moscow

What I am going to show

Given a known mass hierarchy (let us assume **$\Delta m^2_{31}>0$**),
and the knowledge on θ_{13}, how the octant of θ_{23} can be probed in neutrino factories ?

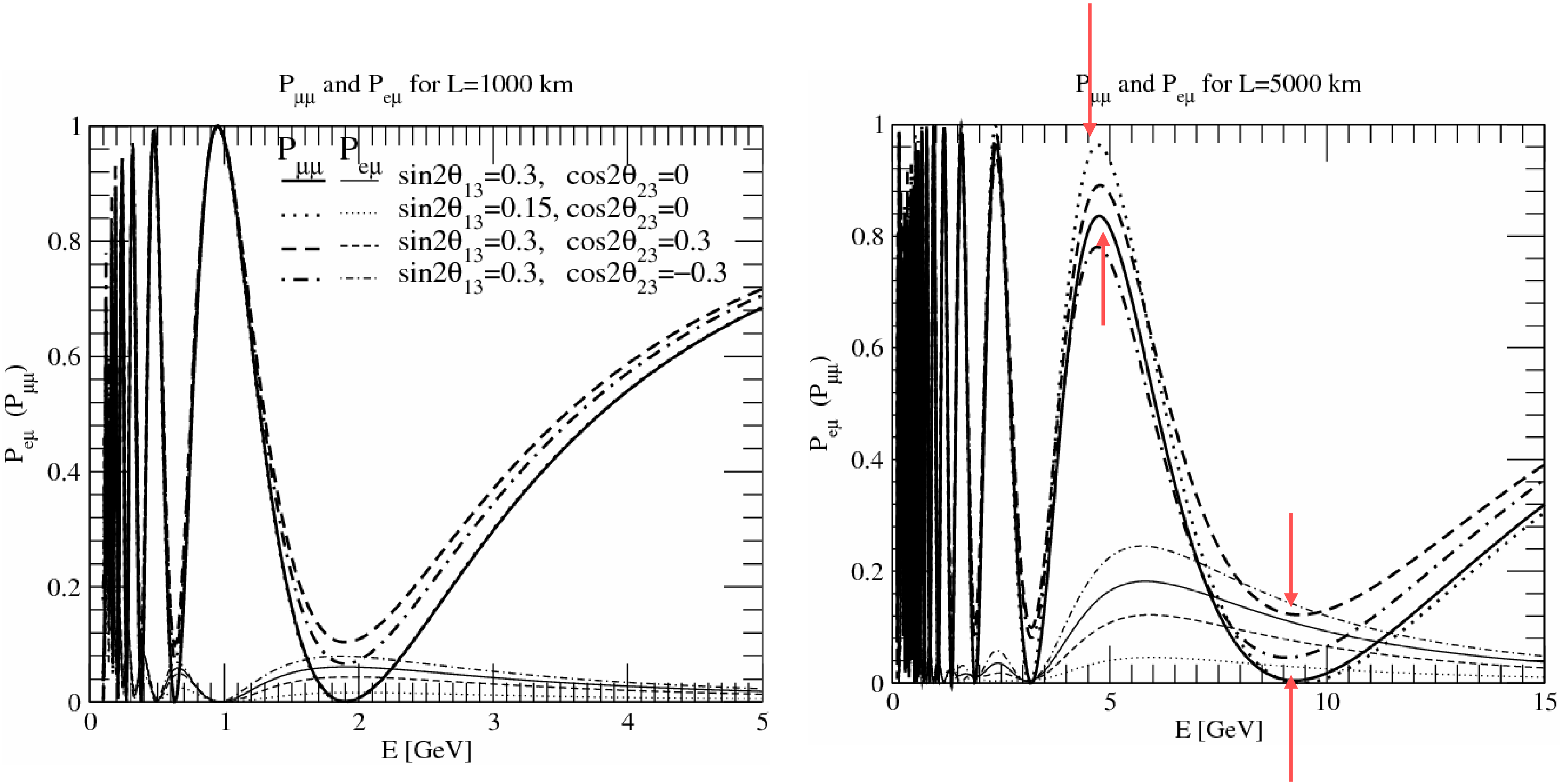
We propose the combined analysis of appearance mode $\nu_e \rightarrow \nu_\mu$ and the disappearance mode $\nu_\mu \rightarrow \nu_\mu$ at a 20 GeV neutrino factory operated at the magic Baseline (7300 km~7600 km).

Previous works on probing θ_{23} octant with neutrino factories:

(1)Combined analysis for all channels include $\nu_e \to \nu_\tau$ at L=3000 km and 7000 km respectively with 50 GeV neutrino factory Donini et al., NPB 2006

(2)Analying only appearance mode $\nu_e \to \nu_\mu$, combining result from L=4000 km and 7300 km (magic baseline). 50 GeV neutrino factory. Gandhi and Winter, PRD 2007

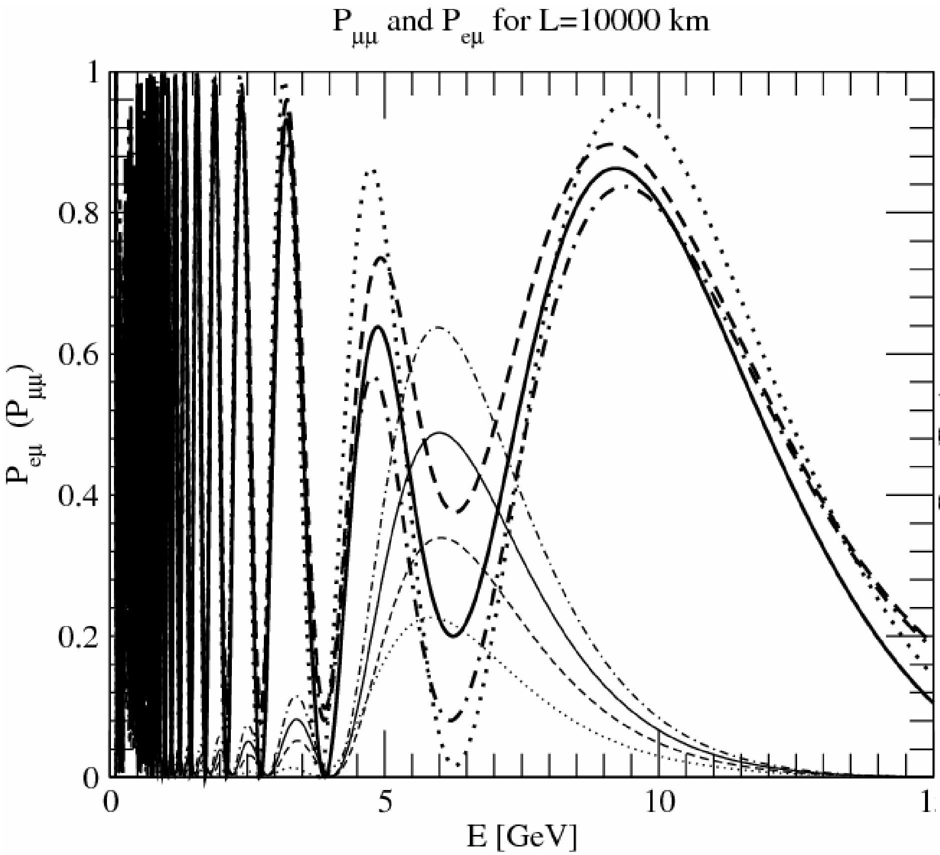
We focus on combining $\nu_e \to \nu_\mu$ and $\nu_\mu \to \nu_\mu$ at the magic baseline in a 20 GeV neutrino factory.
Lower muon detection efficiency, but complementarities between two modes significantly improve the sensitivity.

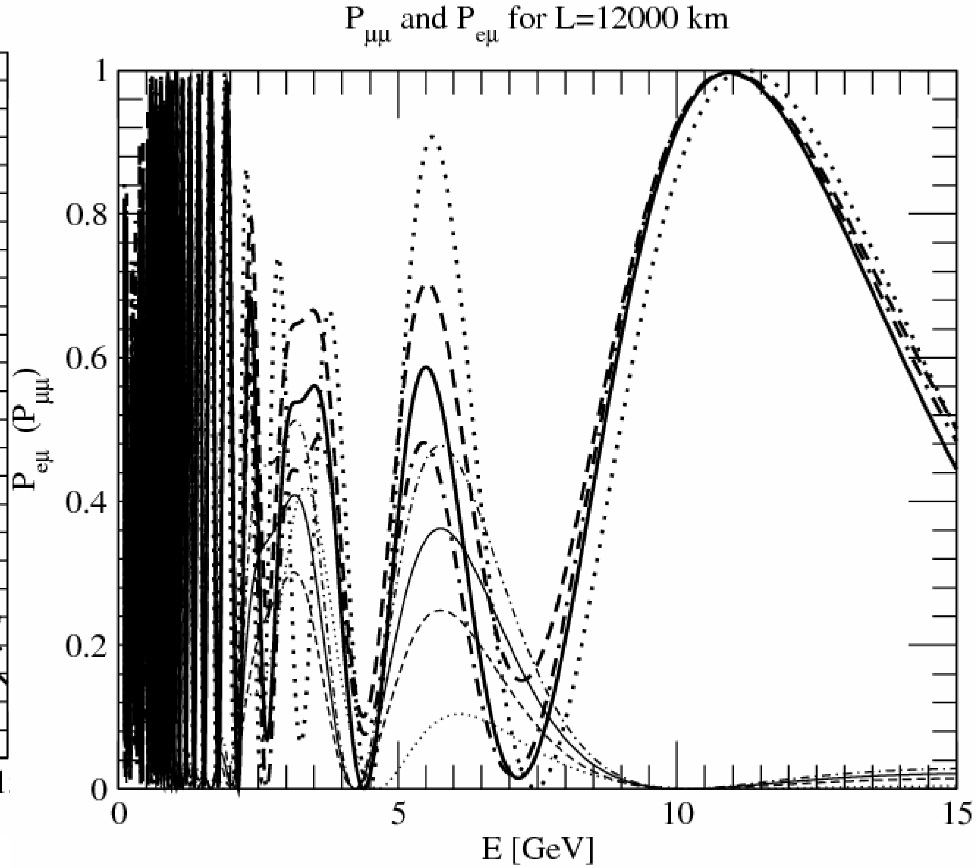


Solid and dashed test θ_{23}-min

Solid and dotted test θ_{13}-max

Numerical results on probabilities

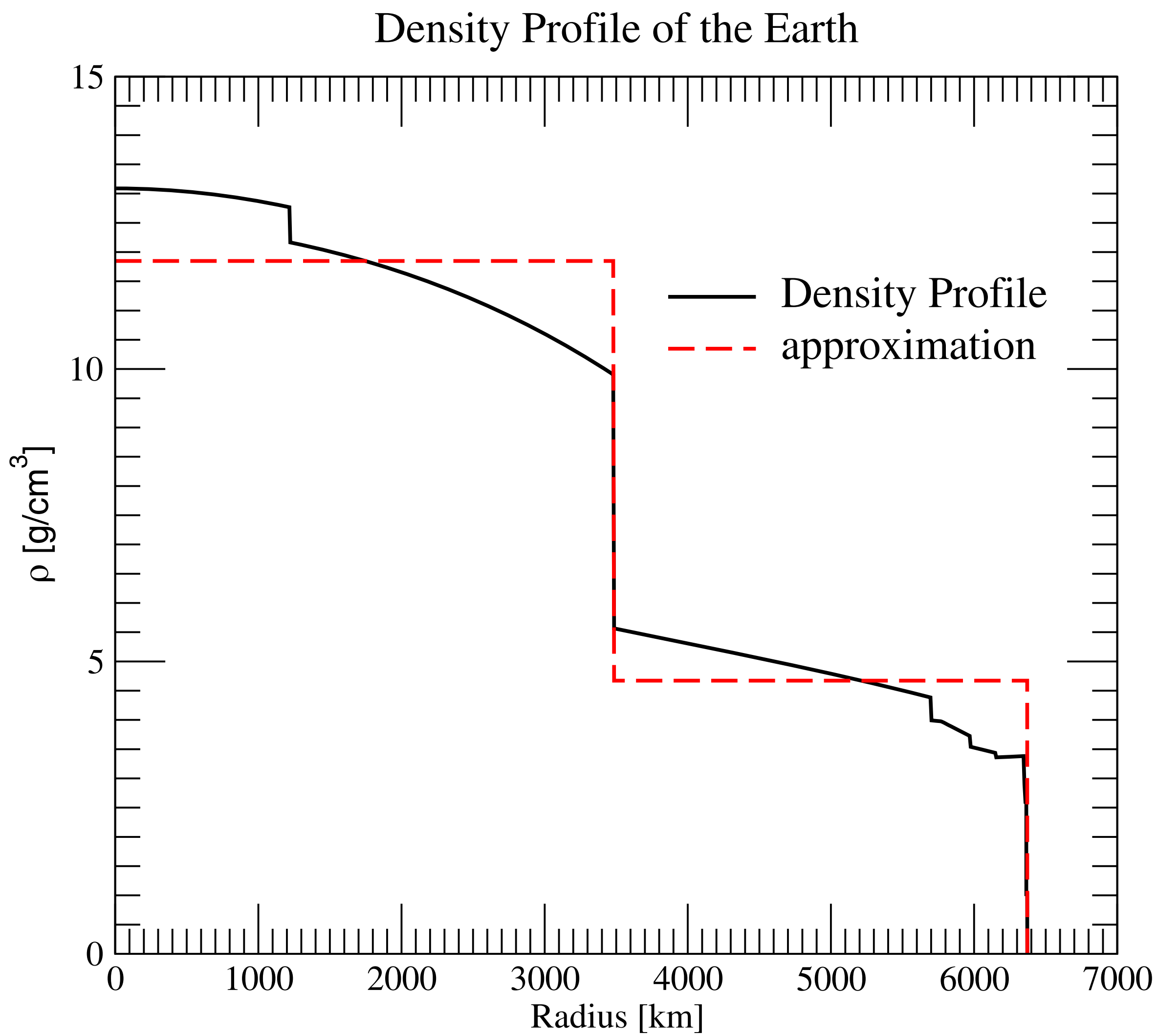




Solid and dashed test θ_{23}
Solid and dotted test θ_{13}

$$\Delta m^2_{31} = 2.4\times10^{-3}\ \mathrm{eV}^2 > 0, \Delta m^2_{21} = 8.2\times10^{-5}\ \mathrm{eV}^2,$$
$$\tan^2\theta_{12} = 0.39, \delta_{cp} = 0.$$

Y. Ashie *et al.,* Phys. Rev. Lett 93, 101801 (2004)
J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, JHEP 0408 (2004)

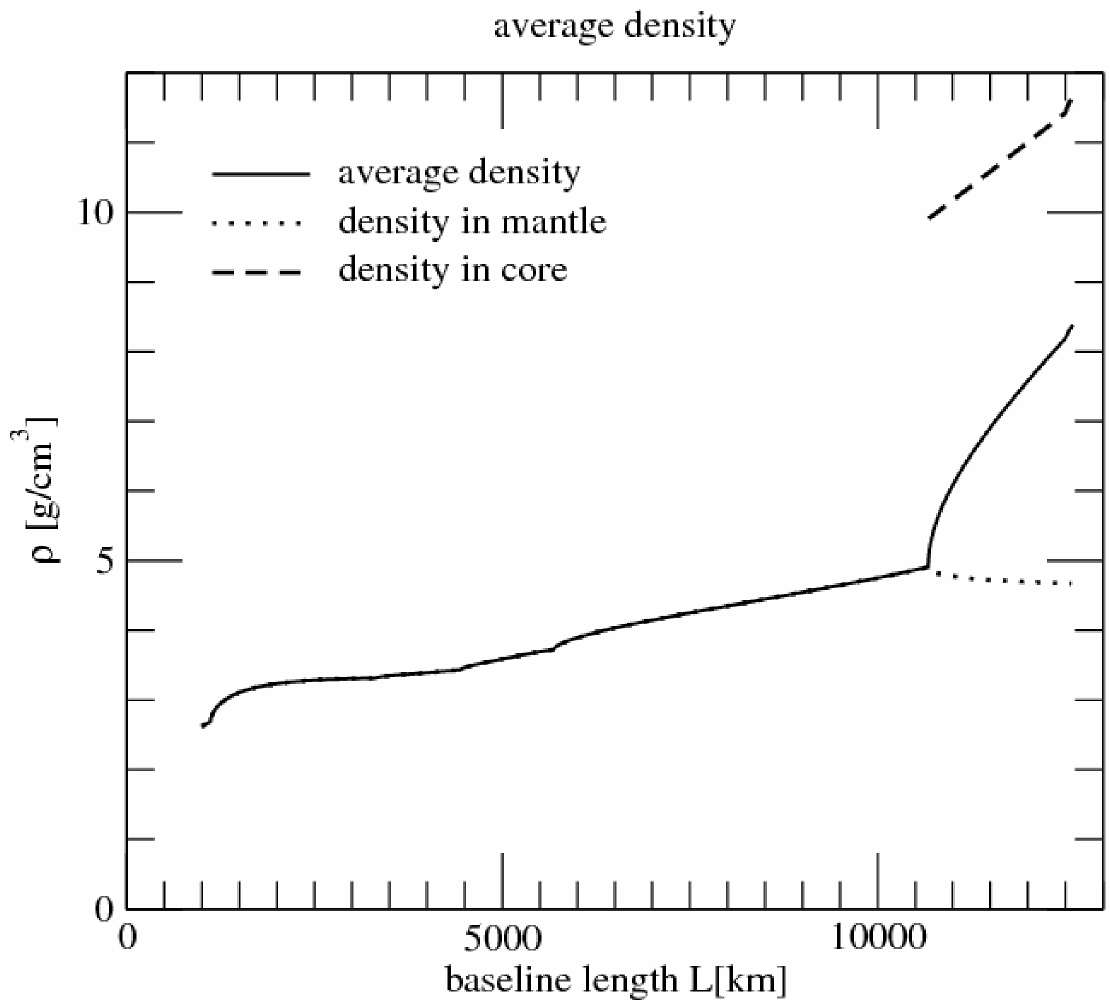


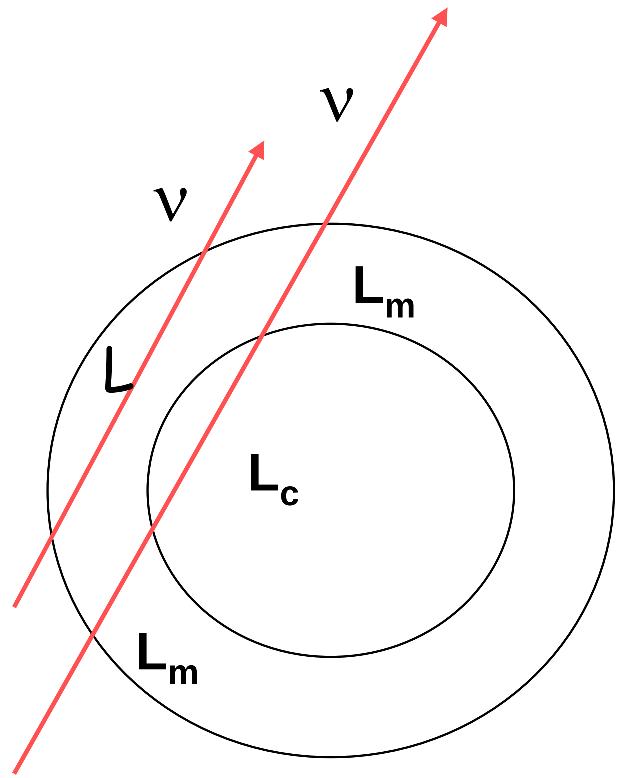
The step functions with
ρ_c=11.85 g/cm^3
ρ_m=4.67 g/cm^3

are good approximation.

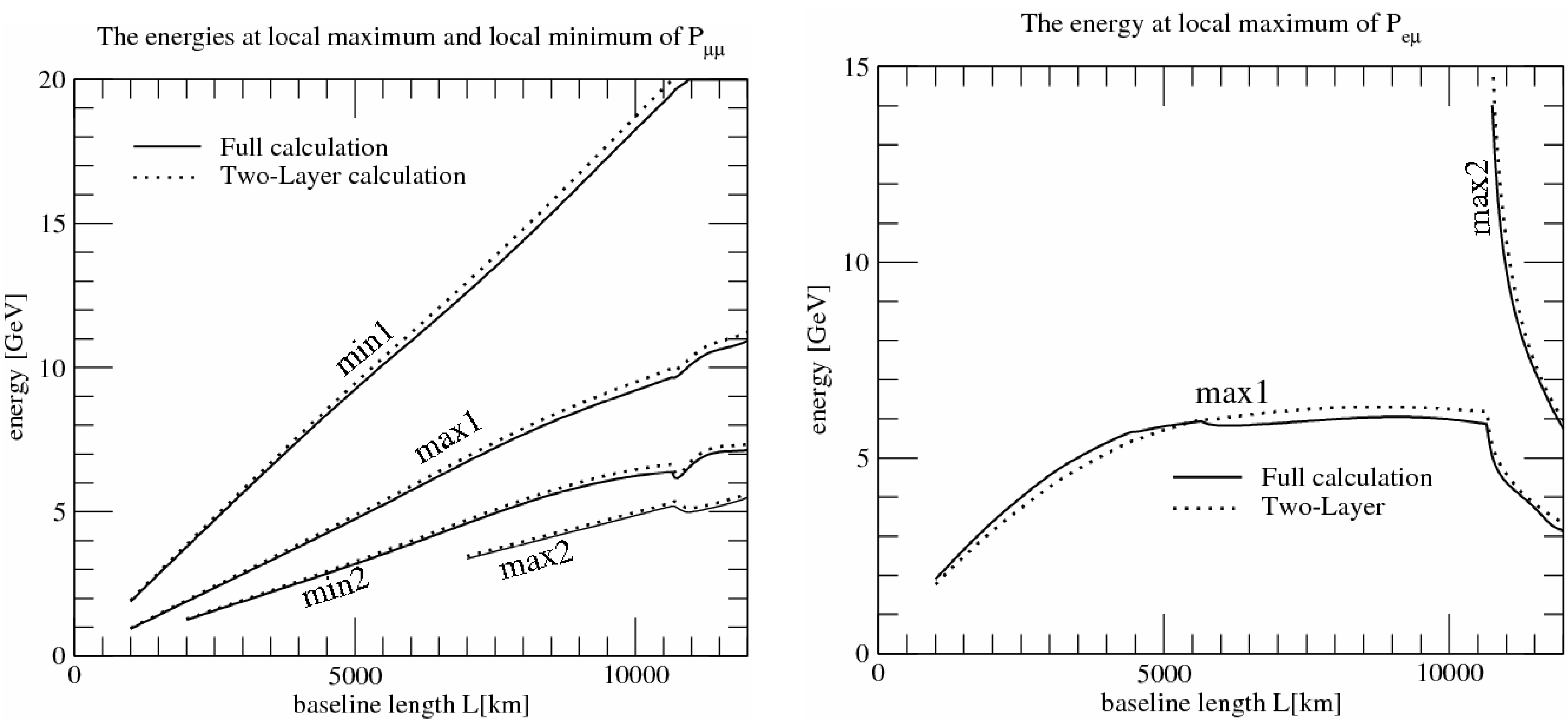
Freund and Ohlsson
hep-ph/9909501

A. M. Dziewonski and D. L. Anderson,
Phys. Earth Planet Int. 25, 297 (1981)





Two density begins at
$L \equiv 2L_m + L_c = 10673$ km



Max2 emerges at 7000 km

An insight from analytic approximations

For a constant medium density:

$$P_{\mu\mu}(y,z) = -\alpha(z)y^2 + (\alpha(z)+\beta(z))y + (1-\beta(z)), P_{e\mu}(y,z) = -(\alpha(z)+\beta(z))(y-1)$$

$$y \equiv \cos 2\theta_{23}, z \equiv \sin 2\theta_{13}, \alpha = a/4 - b - c, \beta = a/4 + b + c,$$

$$a = \sin^2 2\theta_{13}^m \sin^2\left(1.27\Delta_{31}^m L/E\right), b = \cos^2 2\theta_{13}^m \sin^2\left(1.27 M_{31}^2 L/E\right),$$

$$c = \sin^2 2\theta_{13}^m \sin^2\left(1.27 m_{31}^2 L/E\right),$$

$$\Delta_{31}^m = \sqrt{\left(\Delta m_{31}^2 \sin 2\theta_{13}\right)^2 + \left(A_e^m - \Delta m_{31}^2 \cos 2\theta_{13}\right)^2}, \text{ with}$$

$$A_e^m = 2\sqrt{2} E G_F N_e.$$

$$M_{31}^2 = \left(\Delta m_{31}^2 + A_e^m + \Delta_{31}^m\right)/2, m_{31}^2 = \left(\Delta m_{31}^2 + A_e^m - \Delta_{31}^m\right)/2.$$

0th order in the series expansion of $\Delta m^2_{21}/\Delta m^2_{31}$

E. K. Akhmedov *et al.*, JHEP 0404, 078 (2004)

The above forms for oscillation probabilities can be generalized to the two-density case:

Enhanced by matter effect

$$P_{\mu\mu}(y,z) = -\alpha(z)y^2 + (\alpha(z)+\beta(z))y + (1-\beta(z)),\ P_{e\mu}(y,z) = -(\alpha(z)+\beta(z))(y-1).$$

Break the degeneracy

$$\alpha(z) = -\frac{1}{4}\left[(u-\cos t)^2 + (v-\sin t)^2\right],$$

$$\beta(z) = \frac{1}{2}\left(1-u^2-v^2\right) + \frac{1}{4}\left[(u-\cos t)^2 + (v-\sin t)^2\right], \text{ with}$$

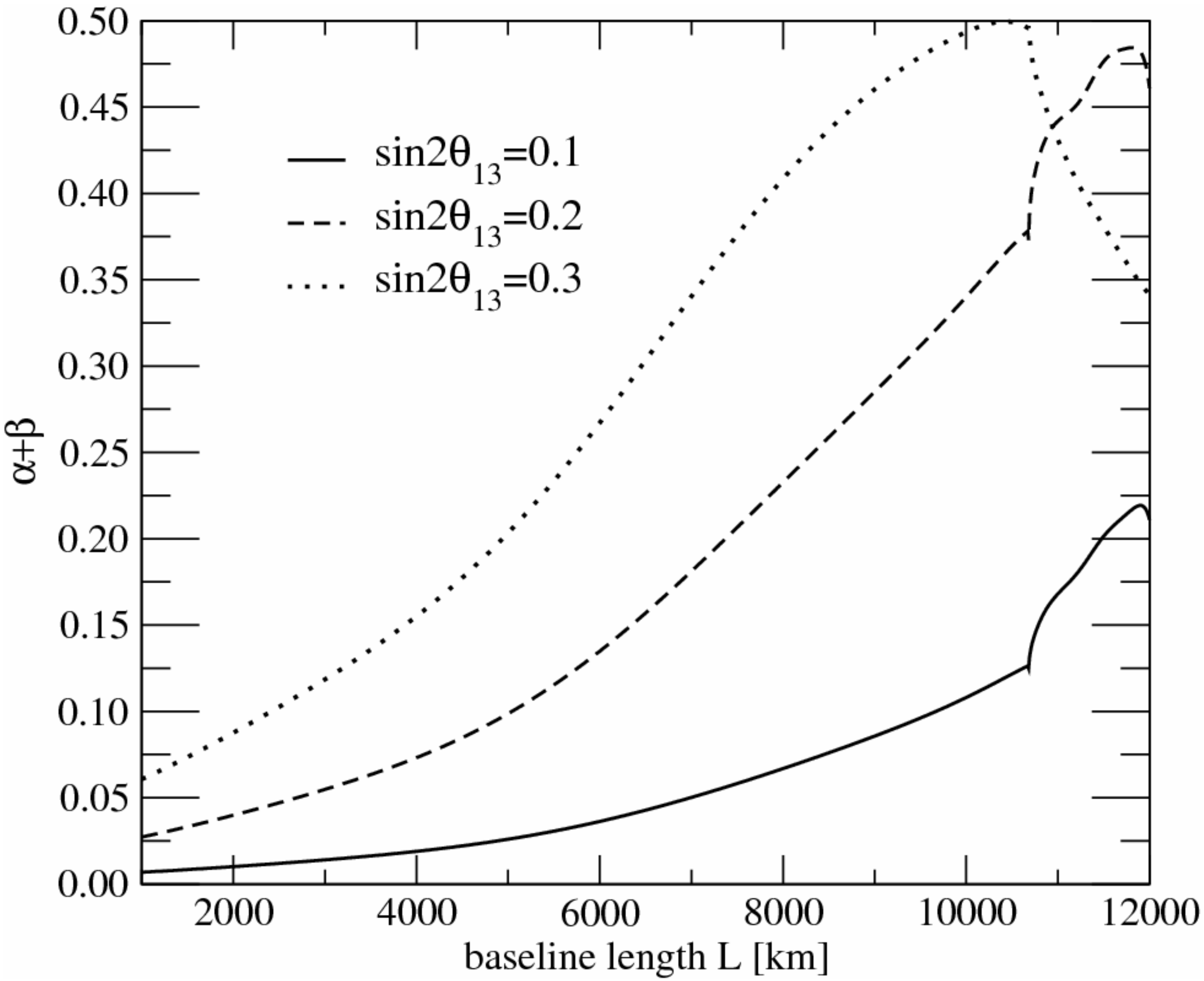
$$u(z) = \cos(2\phi^m)\cos(\phi^c) - \cos(2\theta_{13}^c - 2\theta_{13}^m)\sin(2\phi^m)\sin(\phi^c),$$

$$v(z) = -\cos(2\theta_{13}^m)\left[\sin(\phi^c)\cos(2\phi^m)\cos(2\theta_{13}^c - 2\theta_{13}^m) + \cos(\phi^c)\sin(2\phi^m)\right] + \sin(2\theta_{13}^m)\sin(\phi^c)\sin(2\theta_{13}^c - 2\theta_{13}^m),$$

$$t(z) = \frac{(M_{13}^2)^m + (m_{13}^2)^m}{4E} \times 2L^m + \frac{(M_{13}^2)^c + (m_{13}^2)^c}{4E} \times L^c, \text{ with}$$

$$\phi^{m(c)} = \frac{\Delta_{31}^{m(c)}}{4E} L^{m(c)}.$$

J. Bernabeu, S. Palomares-Ruiz, A. Perez and S.T. Pecov, Phys. Lett. B 531, 90 (2002)



$\alpha + \beta$ indicates the size of matter effect

Maximal value at each distance

Magic baseline

Barger, Marfatia and Whisnet, PRD 2003
Huber and Winter, PRD 2003
Smirnov, hep-ph/0610198

$$P(\nu_\mu \to \nu_e) = |\cos\theta_{23} A_S e^{i\delta} + \sin\theta_{23} A_A|^2$$

A_S: solar amplitude depends on solar oscillation parameters Δm^2_{21}, θ_{12}

A_A: atmospheric amplitude depends on oscillation parameters Δm^2_{31}, θ_{13}

For constant medium density

$$A_S = \sin 2\theta^m_{12} \sin \frac{\phi^m_S}{2}$$

with

$$\phi^m_S \equiv \frac{2\pi L}{l_m}$$

l_m: oscillation length in medium

$L = l_m$, $A_S = 0$, no effect from CP phase δ

In the large energy limit, i.e., the neutrino energy is far beyond the resonant energy,

$$l_m \to l_0 = \frac{2\pi}{\sqrt{2} G_F N_e}$$

l_0=7600 km by taking the Earth average density.

In careful analysis, the magic baseline is 7300~7600 km

The muon neutrino event spectrum

$$\frac{dn_f}{dE'} = N \int dE \times \phi_i(E) \times P_{\nu_i \to \nu_f}(E) \times \sigma_f(E) \times R_f(E,E') \times \epsilon_f(E'),$$

Energy resolution function

Detection Efficiency

Interaction cross section

P. Huber, M. Lindner and W. Winter NPB 2002

$$N = n_y \times 10^9 \times N_A \times M_d,$$

n_y=4 years of running, same for μ^+ and μ^- modes

M_d=50 kilotons magnetized iron detector

The initial flux of ν_e and ν_μ

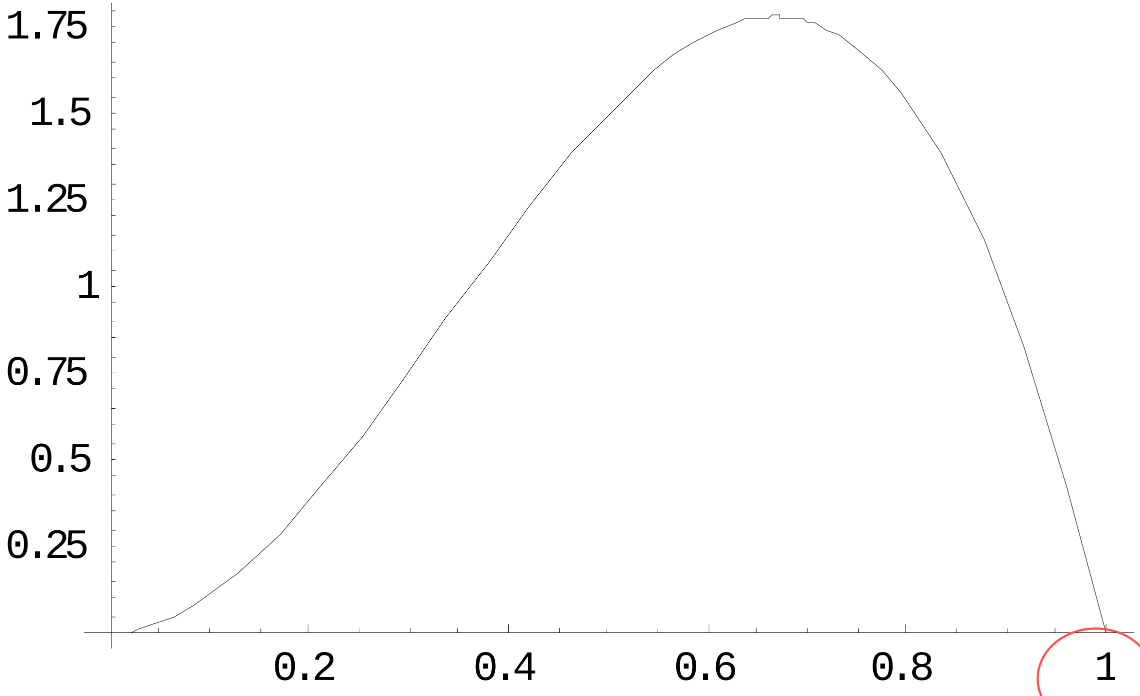
$$\phi_i(E) = \left(\frac{N_\mu E_\mu}{\pi m_\mu^2 L}\right) g_i(E/E_\mu),$$

E_μ=20 GeV

We take $N_{\mu^+} = N_{\mu^-} = 5.3 \times 10^{20}$.

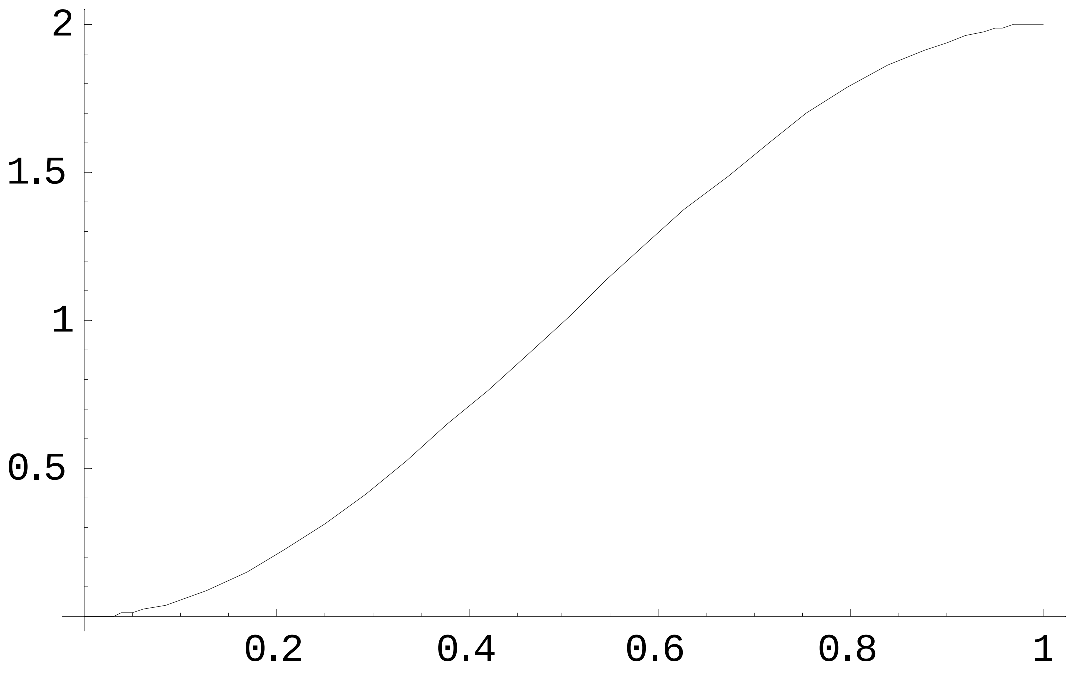
$$g_{\nu_e}(x) = g_{\bar{\nu}_e}(x) = 12x^2(1-x)$$

$$g_{\nu_\mu}(x) = g_{\bar{\nu}_\mu}(x) = 2x^2(3-2x)$$



Initial ν_e spectrum

20 GeV



Initial ν_μ spectrum

Muon neutrino detection efficiency

$$\epsilon_f(E') = \frac{\eta_f}{4}\left(\frac{E'}{4} - 1\right)$$

Rise up to *E'* =20 GeV

E' is the detected muon neutrino energy
in the GeV unit

$$\eta_f = 0.35 \text{ for oscillation modes } \nu_e \rightarrow \nu_\mu \text{ and } \bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$$

For *E'* =10 GeV, η_f=13%

$$\eta_f = 0.45 \text{ for oscillation modes } \bar{\nu}_e \rightarrow \bar{\nu}_\mu \text{ and } \nu_\mu \rightarrow \nu_\mu$$

Energy resolution function

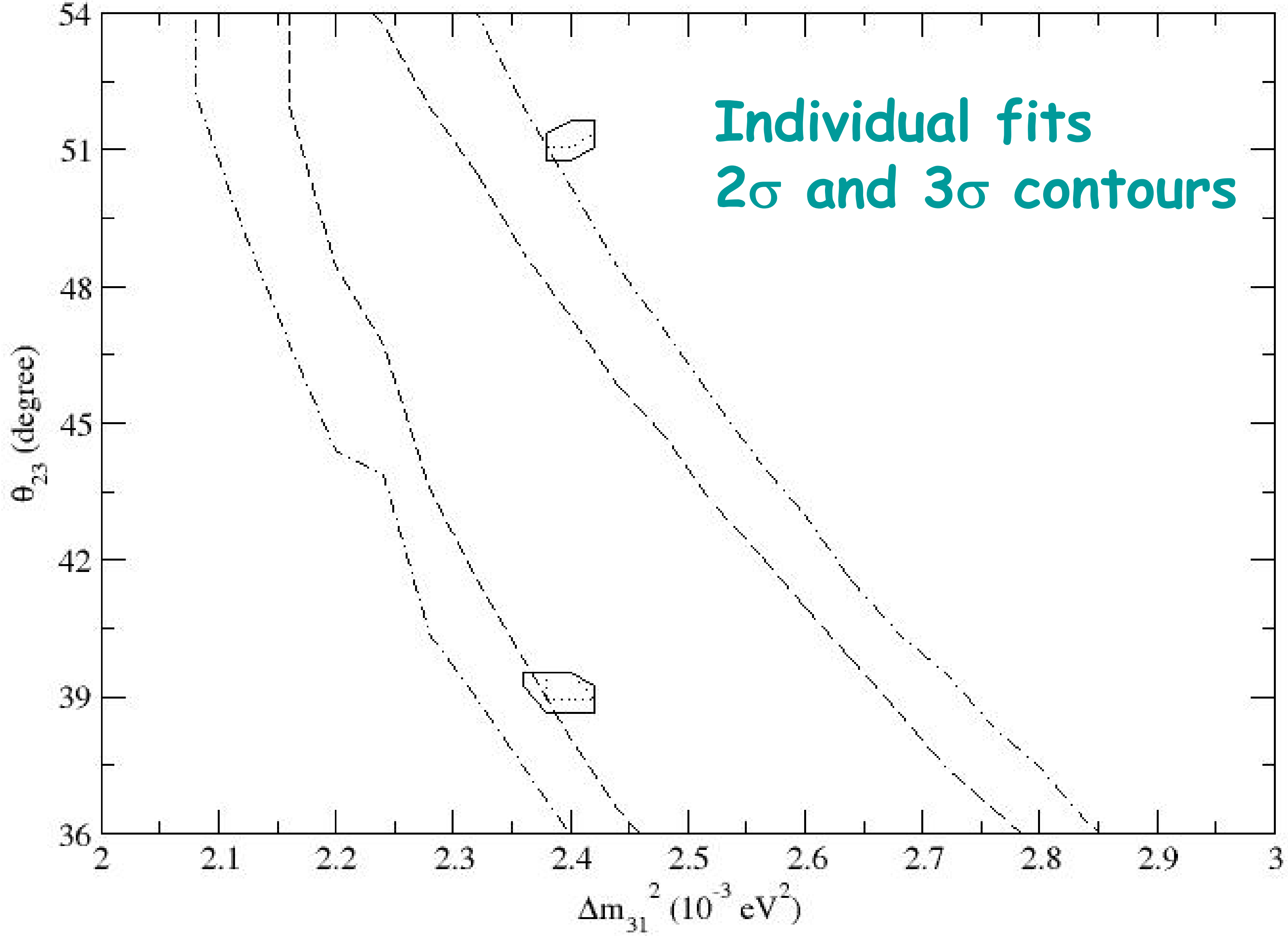
$$R_f(E, E') = \frac{1}{\sqrt{2\pi\lambda_f^2}} \exp\left[-\frac{(E-E')^2}{2\lambda_f^2}\right]$$

E is the true energy
E' is the detected energy

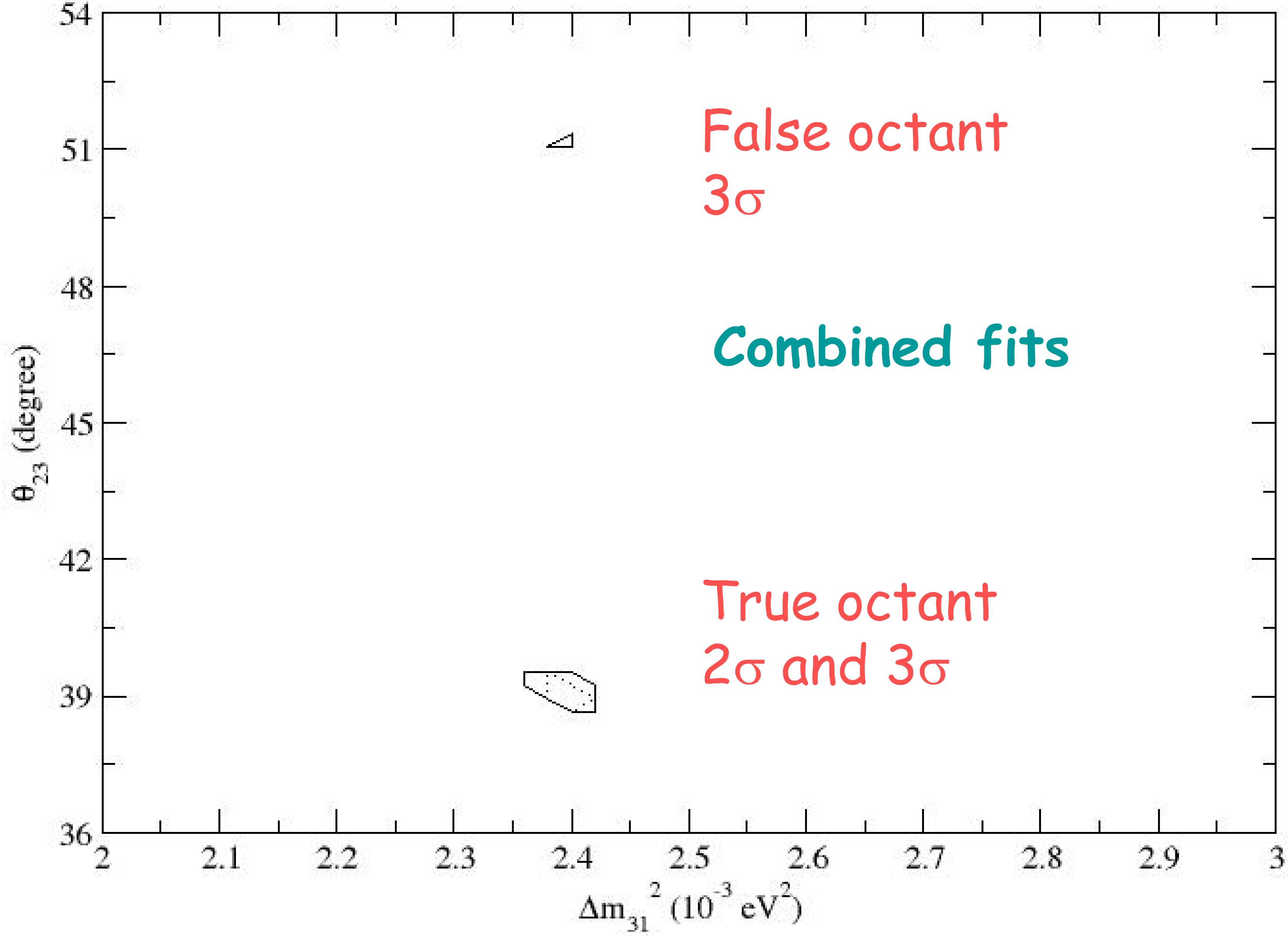
We take $\lambda_f = 0.15E$

Use 1 GeV bin to fit the final muon neutrino spectra

$\sin^2 2\theta_{13}=0.01, \cos 2\theta_{23true}=0.2$ ($\theta_{23true}=39.2^0$)

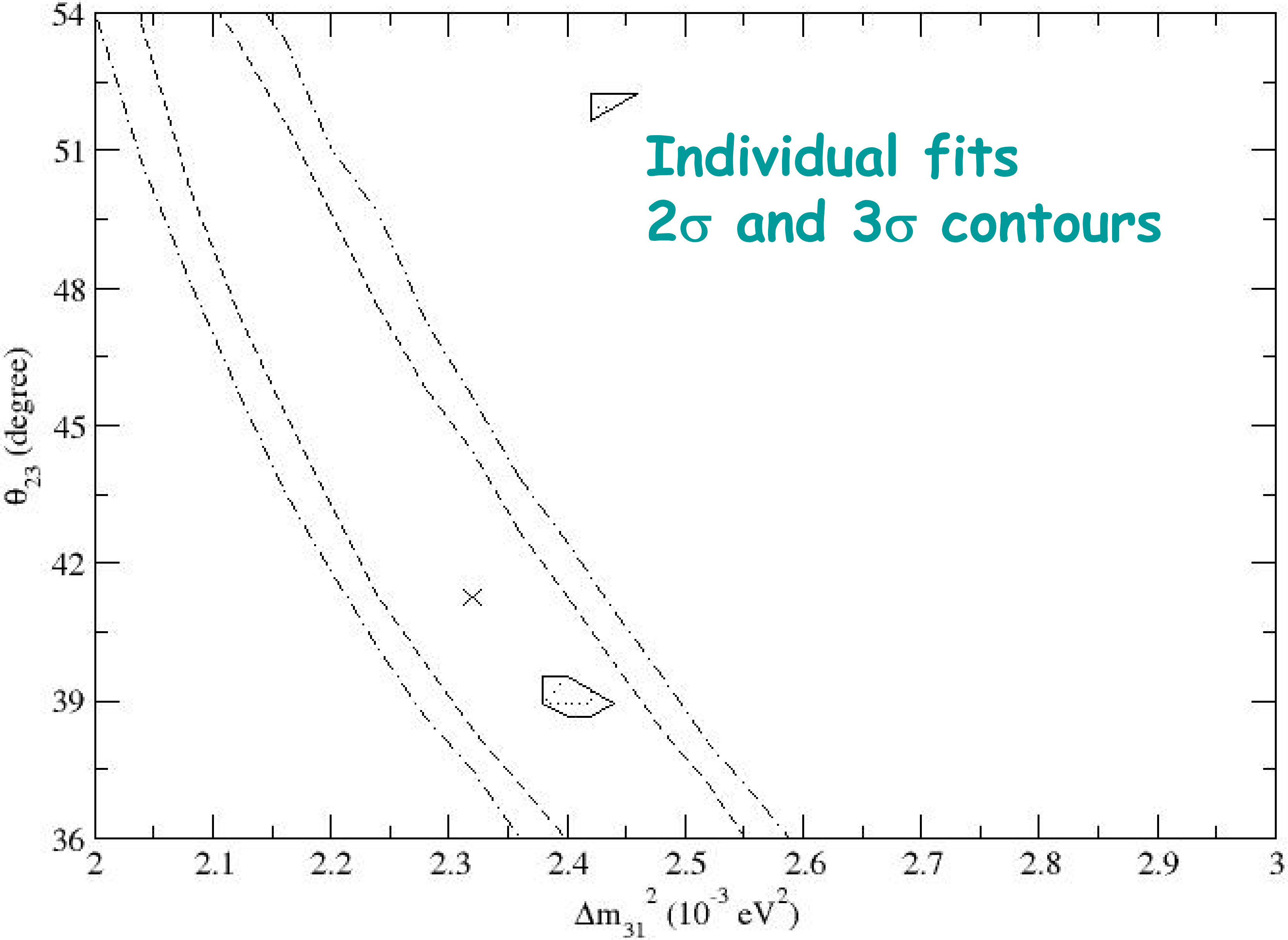


$\Delta m_{31}{}^2{}_{true}=2.4\times10^{-3}$ eV^2

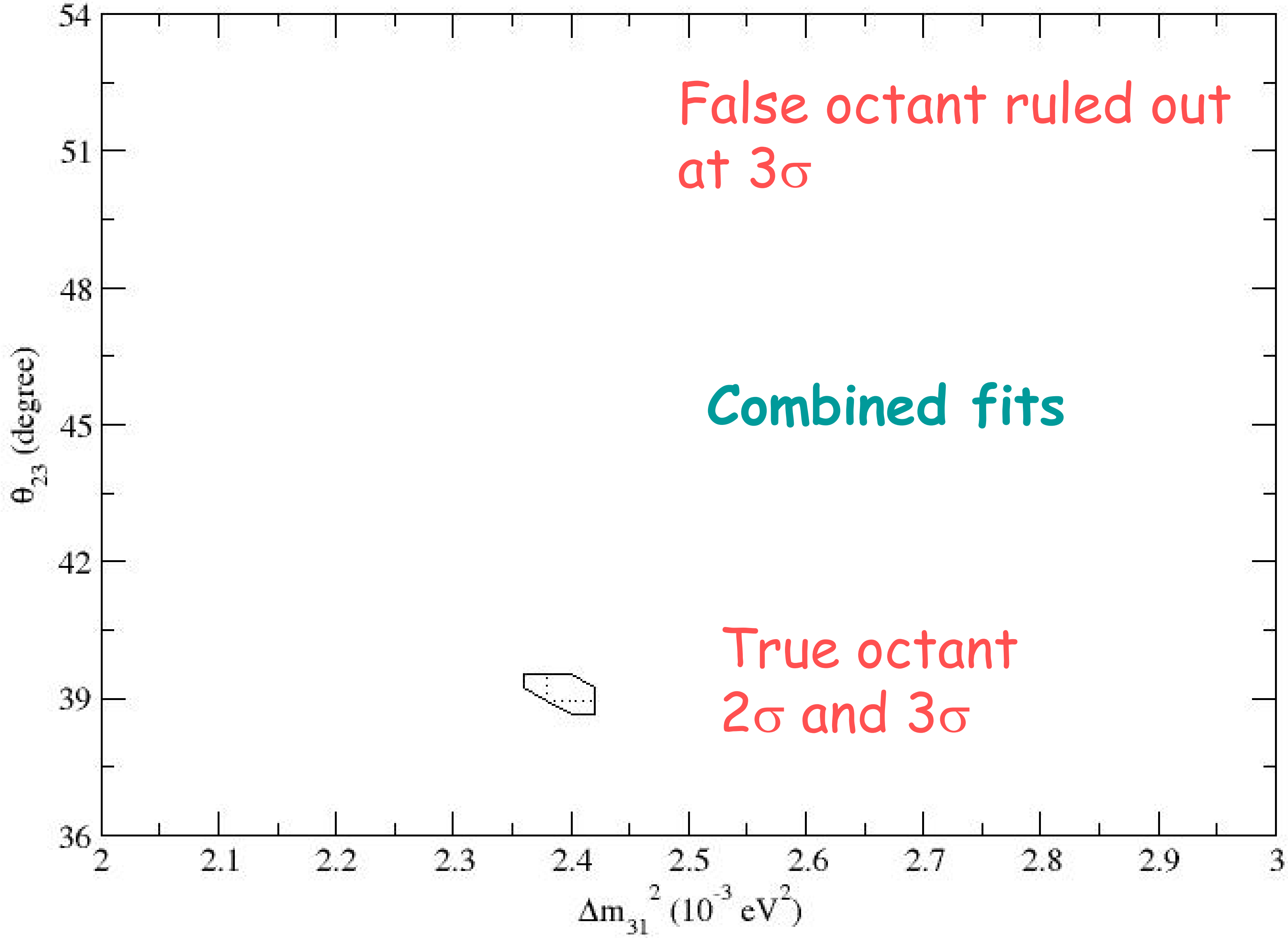


False octant
3σ
Combined fits
True octant
2σ and 3σ
θ23 (degree)
Δm31² (10⁻³ eV²)
54
51
48
45
42
39
36
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3

$\sin^2 2\theta_{13}=0.04, \cos 2\theta_{23true}=0.2$ ($\theta_{23true}=39.2^0$)



$\Delta m_{31}^2{}_{true}=2.4\times10^{-3}$ eV^2



False octant ruled out at 3σ
Combined fits
True octant 2σ and 3σ
θ_{23} (degree)
Δm_{31}^2 (10^{-3} eV^2)
54
51
48
45
42
39
36
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3

Conclusions

- We review the matter effects in disappearance and appearance neutrino oscillation probabilities. These matter effects are crucial to resolve the octant of mixing angle θ_{23}.
- We propose to resolve this octant with a 20 GeV neutrino factory operating on the magic baseline.
- We perform a combined fit to $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_\mu$ oscillation data obtained from 4 years of μ^+ decays and 4 years of μ^- decays where 5.3×10^{20} muons decay in each year. The target is a 50 kiloton magnetized iron detector.

- Assuming $\cos 2\theta_{23true}$=0.2, $\Delta m_{31}{}^2{}_{true}$=$2.4\times10^{-3}$ eV^2, the combined best fit to appearance and disappearance modes (taking into account statistical fluctuations) recover the above parameter values for both $\sin^2 2\theta_{13}$=0.04 and $\sin^2 2\theta_{13}$=0.01. In the former case, the false octant solution is ruled out at 3σ while it is ruled out at 2σ in the latter case. For $\sin 2^2\theta_{13}$=0.09, the false octant solution is ruled out at 3σ with the disappearance mode alone.
- A more complete analysis by varying the true value of θ_{23} is in progress.