Neutrino Asymmetry and Cosmological Birefringence

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Outline

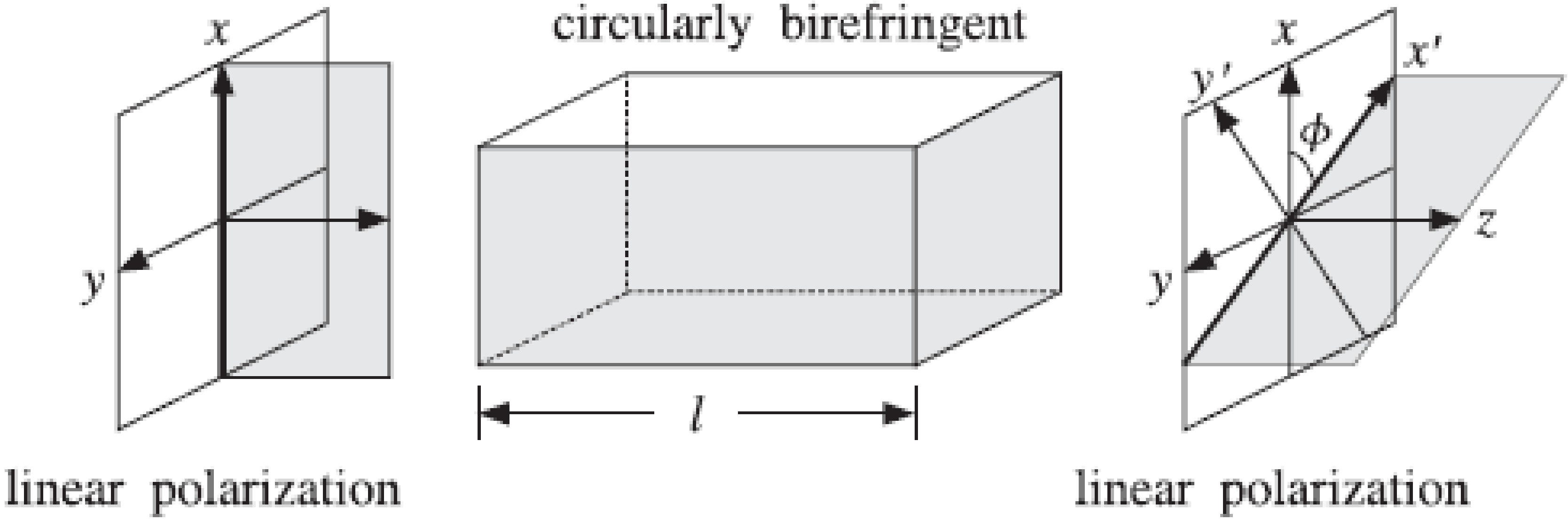
- Cosmological Birefringence
- Previous Theoretical Models
- Our Work - Chern-Simons term with Neutrino Asymmetry
- Summary

Cosmological Birefringence

- About ten years ago Nodland and Ralston made an analysis on data of synchrotron radiation from galaxies and quasars.
  - » Nodland and Ralston, PRL 78 (1997) 3043
- They extracted an additional rotation of position angle of the polarization plane from Faraday rotation, which is wavelength-independent, referred to the cosmological birefringence.

$$\Delta\alpha = C\lambda^2 + \chi$$

- However, it has been shown that there is no statistically significant signal present at that time.
  - » Carroll & Field, PRL 79(1997) 2934 and Eisenstein & Bunn, PRL 79(1997) 1957
- Ni pointed out that the rotation angle can be constrained at the level of 10^{-1} by data of the WMAP due to the correlation between the polarization and temperature
  - » W.T. Ni, Chin. Phys. Lett. 22, 33 (2005)



Picture taken from
http://www.ece.rutgers.edu/~orfanidi/ewa/

- Feng et al used the combined data of the WMAP and the 2003 flight of BOOMERANG for the CMB polarization to constrain the rotation angle and concluded that a nonzero angle is mildly favored. $\Delta\alpha = -6.0^{+4.0}_{-4.0}{}^{+3.9}_{-3.7}$ deg for 1 σ , 2 σ errors.
  - » Feng et al, PRL 96, 221302 (2006)

- The result is less supported by general quintessence model.
  - » G.Liu, S.Lee and K.W.Ng, PRL97, 161303 (2006)

- P. Cabella et al used wavelet analysis of WMAP3 data to derive the constraint $\Delta\alpha = -2.5 \pm 3.0$ and $\Delta\alpha = -2.5 \pm 6.0$ for 1 σ , 2 σ errors.
  - » P. Cabella et al, arXiv:0705.0810 [astro-ph]

Previous Theoretical Models

- This unknown anisotropy can be explained by a modification of Maxwell theory: two different approaches.

a. Ni first found the most general effective interaction for electromagnetic and gravitational field to test the Einstein equivalence principle

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\mathcal{EM}} + \mathcal{L}_{\mathcal{N}} \\ &= -\frac{1}{4}\sqrt{g}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sqrt{g}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\end{aligned}$$

Here ϕ is a dimensionless gravitational scalar

» W.T. Ni, PRL 38, 301 (1977)

b. Alternatively Carroll, Field and Jackiw studied the modified Maxwell theory by adding a Chern-Simons-like term

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\mathcal{EM}} + \mathcal{L}_{\mathcal{CS}} \\ &= -\frac{1}{4}\sqrt{g}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sqrt{g}p_{\mu}A_{\nu}\tilde{F}^{\mu\nu}\end{aligned}$$

» Carroll, Field and Jackiw, PRD 41,1231 (1990)

Here p_{μ} is a constant four-vector with dimension of mass

Notes

– This term is not gauge invariant, but it makes a gauge-invariant contribution to the action.

– It is also a CPT and Lorentz violation effective operator

» S. Coleman and S.L. Glashow, PLB405, 249 (1997)

- We can see that $\mathcal{L}_{CS} = \mathcal{L}_{\mathcal{N}}$ by taking $p_\mu = -2\nabla_\mu \phi$ since the field equations are not affected by a total derivative in action.
  - » Carroll and Field, PRD 43,3789 (1991)

- The field equations become

$$\nabla_\mu F^{\mu\nu} = p_\mu \tilde{F}^{\mu\nu}$$

$$\nabla_\mu \tilde{F}^{\mu\nu} = 0$$

- $p_\mu \tilde{F}^{\mu\nu}$ acts as an extra electric current.

- We can write down the Maxwell equations in component ($\vec{p}=0$)

$$\frac{\partial}{\partial\eta}(R^2\mathrm{E}) - \vec{\nabla}\times(R^2\mathrm{B}) = p_0(R^2\mathrm{B})$$

$$\nabla\cdot\mathbf{E}=0$$

$$\frac{\partial}{\partial\eta}(R^2\mathbf{B})+\nabla\times(R^2\mathbf{E})=0$$

$$\nabla\cdot\mathbf{B}=0$$

Here we work on the Robertson-Walker metric

$ds^2=R^2(\eta)(-d\eta^2+d\mathbf{x}^2)$ and use the convention

$$F^{\mu\nu}=R^{-2}\begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

- The wave equation is

$$\frac{\partial^2}{\partial \eta^2}(R^2 \mathrm{B}) - \nabla^2 (R^2 \mathrm{B}) = -p_0 \nabla \times (R^2 \mathrm{B})$$

- We can see it has a solution of the form $R^2 \mathbf{B}(\mathbf{x}, \eta) = e^{-i\mathbf{k}\cdot\mathbf{x}} R^2 \mathbf{B}(\eta)$

- For simplicity, assume the wave vector k is along the x axis and define $F_{\pm}(\eta) = R^2 B_{\pm}(\eta) = R^2 (B_y \pm i B_z)$

- Then the wave equation becomes

$$\frac{d^2}{d\eta^2} F_{\pm} + (k^2 \pm p_0) F_{\pm} = 0$$

- If we further assume $p_0/k \ll 1$, we can use WKB method to get the solution

$$B_{\pm}(x,\eta)=e^{-ikx}R^{-2}F_{\pm}(\eta)=R^{-2}e^{i\sigma_{\pm}}$$

Here $$\sigma_{\pm} = k(\eta - x) \pm \frac{1}{2}\int p_0 d\eta - \frac{1}{8}\int p_0^2 d\eta + O(k^{-2})$$

- Therefore, the rotation of the position angle is

$$\Delta\alpha = \frac{1}{2}(\sigma_+ - \sigma_-) = \frac{1}{2}\int p_0 d\eta$$

Chern-Simons current with Neutrino Asymmetry

»arXiv:0706.0080 (astro-ph)

- Now we consider the case

$$\mathcal{L} = \mathcal{L}_{\mathcal{EM}} + \mathcal{L}_{\mathcal{CS}}$$
$$= -\frac{1}{4}\sqrt{g}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sqrt{g}p_\mu A_\nu \tilde{F}^{\mu\nu} \qquad p_\mu = \frac{\beta}{M^2}j_\mu$$

Neutrino current: $j_\mu = \bar{\nu}\gamma_\mu\nu \equiv (j^0_\nu, \vec{j_\nu})$ and $j^0_\nu = \Delta n_\nu \equiv n_\nu - n_{\bar{\nu}}$

- In general, the interaction term $\mathcal{L}_{CS} = -\frac{1}{2}\sqrt{g}\frac{\beta}{M^2}j_\mu A_\nu \tilde{F}^{\mu\nu}$ is not gauge invariant under transformation $A_\nu \to A_\nu + \partial_\nu \chi$ since $\Delta\mathcal{L}_{CS} = \frac{\beta}{4M^2}\chi\tilde{F}^{\mu\nu}(\partial_\nu j_\mu - \partial_\mu j_\nu)$ is not vanishing in general

- To fix this problem, we can reformulate our interaction by introducing a Stuckelberg field $S^{\mu\nu}$.

$$\mathcal{L}' = \mathcal{L}_{\mathcal{EM}} + \mathcal{L}'_{\mathcal{CS}}$$
$$= -\frac{1}{4}\sqrt{g}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\sqrt{g}j_{\mu}(A_{\nu}\tilde{F}^{\mu\nu} + \partial_{\nu}S^{\mu\nu})$$

where $S^{\mu\nu}$ is antisymmetric in indices.

- Now the gauge invariant is satisfied by acquiring a gauge transformation of $S^{\mu\nu}$

- It is also interesting to note that above interaction might originate from the low energy effective theory in superstring theory.
  - » W.F. Chen, Private communication

For example, the role of Stuckelberg field $S^{\mu\nu}$ is played by Kalb-Ramond field $B_{\mu\nu}$ with the relation $S^{\mu\beta} = \epsilon^{\mu\beta\sigma\rho} B_{\sigma\rho}$

$$\frac{1}{16}\sqrt{2}\kappa\phi^{-3/4}\bar{\chi}^a\Gamma^{MNP}\chi^a H_{MNP} \qquad \longleftarrow \qquad \text{Eq. } (13.1.42)$$

  - » Green, Schwarz and Witten, Superstring Theory, Vol. 2, Cambridge University Press, 1987

- Then the change in the position angle is

$$\Delta\alpha = \frac{1}{2}\frac{\beta}{M^2}\int \Delta n_\nu(t)\frac{dt}{R(t)}$$

- To have a neutrino-antineutrino asymmetry, there would exist a chemical potential μ_ν

- It is known that for a lepton flavor, the asymmetry is given by

$$\eta_\ell = \frac{n_\ell - n_{\bar{\ell}}}{n_\gamma} = \frac{1}{12\zeta(3)}\left(\frac{T_\ell}{T_\gamma}\right)^3 (\pi^2\xi_\ell + \xi_\ell^3)$$

$\xi_\ell \equiv \mu_\ell / T_\ell$ is the degeneracy parameter

- The lepton asymmetry in our Universe (if exists) resides in neutrinos because of the charge neutrality.
  - » P.D. Serpico and G.G. Raffelt, PRD 71, 127301(2005)

- And the neutrino asymmetry depends only on the electron-neutrino degeneracy parameter ξ_{ν_e} since neutrinos reach approximate chemical equilibrium before BBN

  » A.D. Dolgov et al, NPB 632, 363(2002)

  » Y.Y.Y. Wong, PRD 66, 025015 (2002)

  » K.N. Abazajian, J.F. Beacom and N.F. Bell, PRD 66, 013008(2002)

- Thus the neutrino asymmetry for a relativistic neutrino, say, electron neutrino is then given by $\eta_{\nu_e} \simeq 0.249\xi_{\nu_e}$

  Here we have assumed $(T_{\nu_e}/T_\gamma)^3 = 4/11$

- We note the current bound on the degeneracy parameter is $-0.046 < \xi_{\nu_e} < 0.072$ for 2σ range of baryon asymmetry

  » P.D. Serpico and G.G. Raffelt, PRD 71, 127301(2005)

- Combining all and use $n_\gamma = 2\zeta(3)/\pi^2 T_\gamma^3$, we obtain

$$\Delta n_\nu \simeq 0.061\xi_{\nu_e} T_\gamma^3$$

- After decoupling, the temperature of photon evolve as

$$T_\gamma = \frac{T_D R_D}{R} = T_{\gamma_{today}}(1+z)$$

- Then the angle $\Delta\alpha = \frac{\beta}{M^2} 0.030\xi_{\nu_e} T_{\gamma_{toady}}^3 \int_0^{z_*} (1+z)^3 \frac{dz}{H(z)}$

for a flat and matter-dominated Universe $H(z) = H_0(1+z)^{3/2}$
at photon decoupling $1 + z_* = (1+z)_{decoupling} \simeq 1100$

- As a result, we get $\Delta\alpha \simeq 4.2 \times 10^{-2}$

with $\beta \sim 1,\ M \sim 10\ TeV$ and $\xi_{\nu_e} \sim \pm 10^{-3}$

Summary

- In the literature, there are two different types of models with different origins to explain the cosmological birefringence: the first one is the coupling between a scalar/pseudo-scalar and electromagnetic field; the second is the coupling between a constant four-vector and a Chern-Simons current.

- We have proposed a new model with CPT-even dimension-six effective interactions which generate the cosmological birefringence.

- We have found that $\Delta\alpha = O(10^{-2})$, which could explain the recent result in Feng et al, PRL 96, 221302 (2006).

- In the future, Planck Surveyor will reach a $\Delta\alpha = O(10^{-2} \sim 10^{-3})$ sensitivity, and future experiment on the CMB polarization would reach $\Delta\alpha = O(10^{-5} \sim 10^{-6})$ sensitivity.

» W.T. Ni, Int. J. Mod. Phys. D 14,901 (2005)