Status of AdS/QCD

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1. AdS/CFT Methods

- effective low energy QCD - lattice QCD;
- RS models;
- models without flavor $\sim N_c$ D-branes;
- models with flavors $\sim N_c$ D-branes $+ N_f$ probe D-branes;
- $Dp - Dq$ intersection branes systems:

  - $D3/D7$

  - $D4/D6 - \overline{D6}$

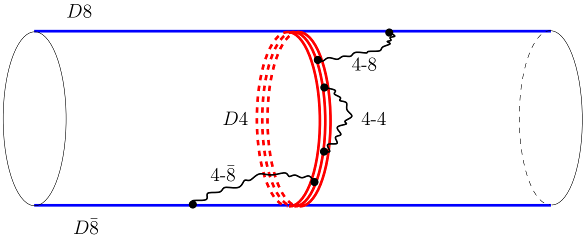
  - $D4/D8 - \overline{D8}, \cdots$

2. *Sakai Sugimoto* model - configuration

- N_f $D8 - \overline{D8}$ pairs in the background of N_c $D4$-branes

	0	1	2	3	(4)	5	6	7	8	9
$D4$	○	○	○	○	○					
$D8 - \overline{D8}$	○	○	○	○		○	○	○	○	○

3. General string description - QCD



- massless modes ⇒ pure QCD

fields	$U(N_c)$	$U(N_f)_L \times U(N_f)_R$
$(4-4) \to A_\mu$	adj.	$(1,1)$
$(4-8) \to q$	fund.	(fund., 1)
$(4-\bar{8}) \to \bar{q}$	fund.	(1, fund.)

- Symmetries in $D4/D8-\overline{D8}$ system.

  1. **supersymmetry:** breaking-by anti-periodic condition for fermions on S^1;
  2. **gauge symmetry:** $SU\left(N_c\right)$
  3. **chiral symmetry:** $U(N_f)_L \times U(N_f)_R$
  4. **internal symmetry:** $SO\left(5\right)$ - ignored.

4. Supergravity description - low energy QCD

- supergravity reliable regime:

small curvature: $\lambda = g_{YM}^2 N_c \gg 1$,

small string coupling: $g_{YM}^4 \lambda = g_{YM}^6 N_c \ll 1$,

$\Rightarrow g_{YM} \to 0, N_c \to \infty, \lambda = g_{YM}^2 N_c \gg 1$ fixed.

strong coupling - low energy behavior of QCD

- The near horizon $D4$-brane solution reads

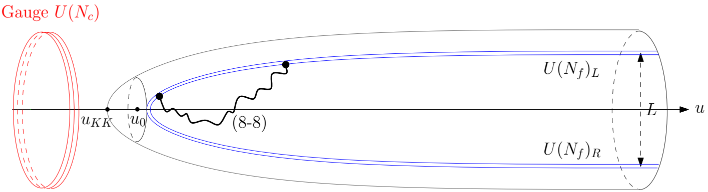
$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left[-dt^2 + \delta_{ij} dx^i dx^j + f(u) d\tau^2\right] + \left(\frac{R}{u}\right)^{3/2} \left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right].$$

$$e^{\phi} = g_s \left(\frac{u}{R}\right)^{3/4}, \quad F_4 \equiv dC_3 = \frac{3N_c}{4\pi}\epsilon_4,$$

$$R^3 = \pi g_s N_c l_s^3, \quad f(u) = 1 - \frac{u_{KK}^3}{u^3} > 0,$$

where ϵ_4 is the volume and

$$\tau \sim \tau + 2\pi M_{KK}^{-1}.$$



- geometric properties:

  1. to avoid a conical singularity at $u = u_{KK}$
     $\Longrightarrow 2\pi M_{KK}^{-1} \equiv \dfrac{4\pi R^{3/2}}{3u_{KK}^{1/2}}$;
  2. $u > u_{KK}$ is bounded from below.

- physical properties:

  1. anti-periodic boundary condition for the fermions

     $\Longrightarrow$ SUSY-breaking;

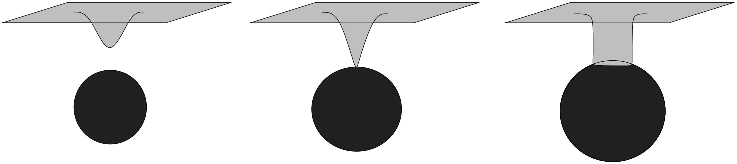
  2. spontaneous chiral breaking:

     $U(N_f)_L \times U(N_f)_R \to U(N_f)_V$

     Nambu-Goldstone bosons $\Longleftrightarrow$ pion.

  3. "color branes" obscure $\Longleftrightarrow$ confinement.

5. Supergravity description - finite temperature



- horizon increases with temperature;
- space-time may finally fall into the black hole at a critical temperature T_c.

- The finite temperature $D4$-brane solution reads $(t \to -it_E)$

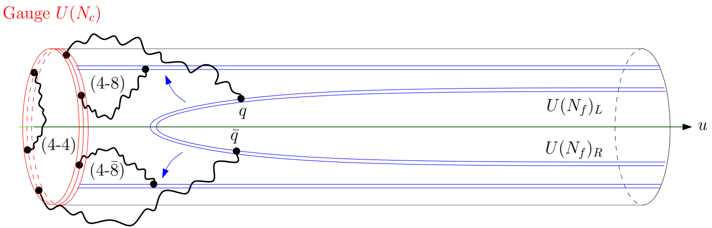
$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left[\tilde{f}(u)\, dt_E^2 + \delta_{ij} dx^i dx^j + d\tau^2\right] + \left(\frac{R}{u}\right)^{3/2} \left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right].$$

$$e^\phi = g_s \left(\frac{u}{R}\right)^{3/4}, \quad F_4 \equiv dC_3 = \frac{3N_c}{4\pi}\epsilon_4,$$

$$R^3 = \pi g_s N_c l_s^3, \quad \tilde{f}(u) = 1 - \frac{u_T^3}{u^3} > 0,$$

where ϵ_4 is the volume and

$$\tau \sim \tau + 2\pi M_{KK}^{-1},\ t_E \sim t_E + \beta.$$

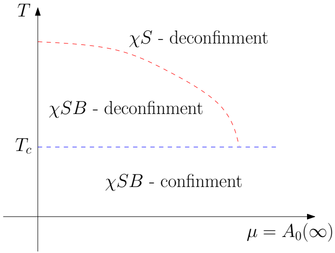


- to avoid a conical singularity at $u = u_T$

$$\Longrightarrow \beta \equiv \frac{4\pi R^{3/2}}{3u_T^{1/2}};$$

- phase transition at $u_T = u_{KK} \Rightarrow T_c = \dfrac{1}{\beta_c} = \dfrac{M_{KK}}{2\pi}$.

- phase transition at $T = T_c$,



	$T < T_c$	$T > T_c$
space-time	near horizon	black hole
confinement	confinement	deconfinement
D8-brane	connected	connected $\leftrightarrow$ disconnected
Chiral symmetry	χSB	$\chi SB \leftrightarrow \chi S$
mesons	stable	stable $\leftrightarrow$ unstable

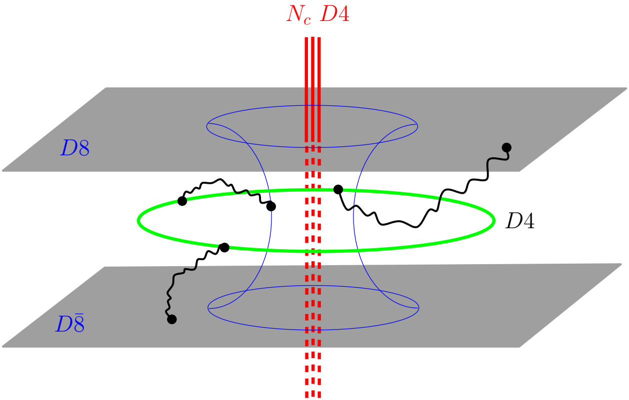
6. Baryons

- solitons of pion effective action, in which baryons appear as solitons called the Skyrmion.

- wrapped $D4$ with N_c fundamental strings sticking onto it.

  Chern-Simon term of the wrapped $D4$-brane

$$S_{CS} = \mu_4 \int C_3 \wedge 2\pi\alpha' f dA = 2\pi\alpha' \mu_4 \int dC_3 \wedge A,$$



N_c $D4$
$D8$
$D4$
$D\bar{8}$

7. Effective Action

- $D8$-brane action

$$S_{D8}^{DBI} = -T \int d^9 x e^{-\phi} \sqrt{-\det\left(g_{MN} + 2\pi\alpha' F_{MN}\right)}$$
$$= \kappa \int d^4 x dz \left[\frac{1}{2} K^{-1/3} F_{\mu\nu}^2 + K F_{\mu z}^2\right] + \mathcal{O}\left(F^3\right),$$

where

$$F = dA,$$
$$K\left(z\right) = 1 + z^2.$$

- fields expansion

$$A_\mu\left(x^\mu, z\right) = \sum_n B_\mu^n\left(x^\mu\right)\psi_n\left(z\right),$$
$$A_z\left(x^\mu, z\right) = \sum_n \varphi^n\left(x^\mu\right)\phi_n\left(z\right),$$

- The EOM of ψ_n reads

$$-K^{1/3}\partial_z\left(K\partial_z\psi_n\right) = \lambda_n\psi_n,$$

and the normalization condition is given by

$$\kappa\int dz K^{-1/3}\psi_n\psi_m = \delta_{nm}.$$

- gauge fixing:

$\underline{\lim_{z\to\pm\infty} A_M = 0:}$

$$A_\mu\left(x^\mu, z\right) = \sum_{n\geq 1} B_\mu^{(n)}\left(x^\mu\right)\psi_n\left(z\right),$$

$$A_z\left(x^\mu, z\right) = \varphi^{(0)}\left(x^\mu\right)\phi_0\left(z\right) + \sum_{n\geq 1}\varphi^n\left(x^\mu\right)\phi_n\left(z\right),$$

$$S_{D8}^{DBI} \sim \int d^4x\left[\frac{1}{2}\partial_\mu\varphi^{(0)}\partial_\nu\varphi^{(0)} + \sum_{n\geq 1}\left(\frac{1}{4}F'^{(n)}_{\mu\nu}F'^{\mu\nu(n)} + \frac{1}{2}\lambda_n^2 B'^{(n)}_\mu\right)\right].$$

where $B'^{(n)}_\mu = B^{(n)}_\mu + \lambda_n^{-1}\partial_\mu\varphi^{(n)}$.

$\underline{A_z = 0:}$ for $U \in U(N_f)_V \subset U(N_f)_L \times U(N_f)_R$

$$A_\mu(x^\mu, z) = U^{-1}\partial_\mu U \psi_+(z) + U\partial_\mu^{-1} U \psi_-(z) + \sum_n B_\mu^n(x^\mu)\psi_n(z).$$

Nambu-Goldstone bosons $U(x^\mu) \longmapsto$ pion field $\Pi(x^\mu)$

$$U(x^\mu) \equiv e^{2i\Pi(x^\mu)/f_\pi}.$$

effective action of the pion - Skyrme action

$$S_{D8}^{DBI}\Big|_{B_\mu^n} = \int dx^4 \left(\frac{f_\pi^2}{4} \mathrm{tr}\left(U^{-1}\partial\mu U\right)^2 + \frac{1}{32e_S^2} \mathrm{tr}\left[U^{-1}\partial\mu U, U^{-1}\partial\nu U\right]^2 \right).$$

8. Phenomenology

- QCD realization,

$$A_\mu = \frac{1}{2f_\pi^2}\left[\Pi, \partial_\mu \Pi\right] + \frac{i}{f_\pi}\partial_\mu \Pi + \sum_{n=1}^{\infty} B_\mu^{(n)} \psi_{2n-1},$$

we identify fields as following,

$\Pi \sim \pi$ (pion),

$B_\mu^1 \sim \rho\,(770)\,,\quad B_\mu^3 \sim \rho\,(1450)\,,\quad B_\mu^5 \sim \rho\,(1700)\,,\cdots$ (vector)

$B_\mu^2 \sim a_1\,(1260)\,,\ B_\mu^4 \sim a_1\,(1640)\,,\cdots$ (axial-vector)

8.1. Vector mesons

- Numerical solve the EOM of ψ_n

$$-K^{1/3}\partial_z\left(K\partial_z\psi_n\right)=\lambda_n\psi_n,\ \kappa\int dzK^{-1/3}\psi_n\psi_m=\delta_{nm}.$$

$$\Rightarrow\lambda_n^{CP}=\rho\left(770\right)/0.67,\quad a_1\left(1260\right)/1.6,$$

$$\rho\left(1450\right)/2.987,a_1\left(1640\right)/4.5,\cdots$$

- Compare to experiments

$$\frac{\lambda_2}{\lambda_1}\simeq\frac{1.6}{0.67}\simeq 2.4\Longleftrightarrow\frac{m^2_{a_1(1260)}}{m^2_\rho}\simeq\frac{\left(1230Mev\right)^2}{\left(776Mev\right)^2}\simeq 2.51$$

$$\frac{\lambda_3}{\lambda_1}\simeq\frac{2.9}{0.67}\simeq 4.3\Longleftrightarrow\frac{m^2_{\rho(1450)}}{m^2_\rho}\simeq\frac{\left(1465Mev\right)^2}{\left(776Mev\right)^2}\simeq 3.56$$

8.2. Massive scalar mesons

- numerical solve the EOM of ρ_n

$$K^{1/3}\left[-\partial_z\left(K\partial_z\rho_n\right)+2\rho_n\right]=\lambda_n'\rho_n,$$
$$\kappa'\int dzK_n^{-1/3}\rho_n\rho_m=\delta_{nm}.$$

$$\Rightarrow \lambda_n'^{CP}=a_0\left(1450\right)/3.3,\cdots$$

- compare to experiments

$$\frac{\lambda_1'}{\lambda_1}\simeq\frac{3.3}{0.67}\simeq 4.9\Longleftrightarrow\frac{m^2_{a_0(1450)}}{m^2_\rho}\simeq\frac{\left(1474Mev\right)^2}{\left(776Mev\right)^2}\simeq 3.61$$

8.3. Other numerical results

- Mass and coupling constants $\left(\kappa \simeq 7.45 \times 10^{-3}\right)$,

n	$m^2_{v^n}$	$\kappa^{-1/2} g_{v^n}$	$\kappa^{-1/2} g_{v^n \pi\pi}$	$m^2_{a^n}$	$\kappa^{-1/2} g_{a^n}$
1	0.669	2.11	0.415	1.57	5.02
2	2.87	9.10	−0.109	4.55	14.4
3	6.59	20.8	0.0160	9.01	28.3
4	11.8	37.1	−0.00408	15.0	46.9

n	$\kappa^{-1/2} g_{\rho v^n v^n}$	$\kappa^{-1/2} g_{\rho a^n a^n}$
1	0.447	0.286
2	0.269	0.257
3	0.252	0.249
4	0.247	0.246

- KSRF (Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin) relation

$$\left.\begin{array}{r} g_\rho = 2 g_{\rho\pi\pi} f_\pi^2 \\ m_\rho^2 = 2 g_{\rho\pi\pi}^2 f_\pi^2 \end{array}\right\} \Rightarrow g_\rho g_{\rho\pi\pi} = m_\rho^2 .$$

- Weinberg sum rules

$$\sum_{n=1}^{\infty} \left(\frac{g_{v^n}^2}{m_{v^n}^2} - \frac{g_{a^n}^2}{m_{a^n}^2} \right) = f_\pi^2, \quad \sum_{n=1}^{\infty} \left(g_{v^n}^2 - g_{a^n}^2 \right) = f_\pi^2 .$$

- Vector meson dominance hypothesis

$$F_\pi = \sum_{n=1}^{\infty} \frac{g_{v^n} g_{v^n \pi\pi}}{m_{v^n}^2} = 1 .$$

9. Summary

- N_f $D8/\overline{D8}$ probe branes in the background of N_c $D4$-branes;
- χSB: $U(N_f)_L \times U(N_f)_R \to U(N_f)$, *NG* bosons $\to$ pion;
- finite temperature: confinement phase transition;
- numerical calculation $\leftrightarrow$ experimental data ($\lesssim$ 20%);
- Pion mass, glueball, technicolor...
- Predictions.