$B \to M\eta^{(\prime)}$ decays in the pQCD factorization approach

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The PPP7 Workshop, NTU, Taipei, June 7-10, 2007.

I present the results of our calculations for the BR and CP-violating asymmetries for some $B/B_s \to M\eta^{(\prime)}$ decays by employing the pQCD factorization approach at leading order.

- Decays considered: $B/B_s \to (\pi, \rho, \omega, \phi, \eta^{(\prime)})\ \eta^{(\prime)}$;
- Nucl.Phys. B738(2006)243;
  Phys.Rev. D73, 074002(2006); Phys.Rev. D75, 014018(2007);
  Phys.Rev. D75, 034017(2007); Phys.Rev. D75, 054033(2007);
  hep-ph/0701146; hep-ph/0704.1027;
- In collaboration with Cai-Dian Lu, Li-bo Guo, Xin Liu, Hui-sheng Wang, Dong-qin Guo and Xin-fen Chen.

1. Introduction and Motivation

With the great progress in both theory and experiments during the past thirty years, nowadays, B physics becomes one of the most active research areas in high energy physics.

⋆ Motivation

- The experimental measurements and theoretical studies of $B \to M_1 M_2$ decays play an important role in the precision test of the SM and in searching for NP beyond the SM.
- The BaBar and Belle collaborations have been collected about 1000 M events of $B\bar{B}$ pair prod. and decays. Many decays, such as

$B \longrightarrow \pi\pi, K\pi, \rho\pi, \cdots$ have been measured with good precision. Much more B meson events ($B_{u,d}, B_s, B_C$ and $b-$hadrons) are expected at the forthcoming LHC experiments.

- In the SM, the decay amplitude can be written as

$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i \, C_i(\mu) < M_1 M_2 | O_i(\mu) | B > . \qquad (1)$$

The Wilson coefficients are known at NLO level, but the TH uncertainties from the evaluation of $< M_1 M_2 | O_i(\mu) | B >$ are still rather large.

- Due to the so-called $B \to K\eta'$ puzzle, the $B \to M\eta^{(\prime)}$ decays have been studied extensively. But we still can not provide a satisfactory explanation for this puzzle, or for the observed patterns of BR's for the four $B \to K\eta^{(\prime)}$ decays or $B \to K^*\eta^{(\prime)}$ decays.

- Our understanding for $B \to M\eta^{(\prime)}$ decays is not as good as other decay modes, because of the special feature of η' meson. We still do not know how large is the possible gluonic component in η' meson ?

⋆ Factorization approaches

In recent years, great progress has been made in evaluating hadronic matrix elements, and the popular factorization approaches include:

- QCD Factorization [BBNS, D.S.Du et al.,];

- pQCD Factorization [H.N. Li, C.D. LÜ, A.I. Sanda, et al.,];

- The soft-collinear effective theory [Bauer, Stewart, Beneke, et al.,].

⋆ Outline of the pQCD Approach

- In pQCD, the k_T is kept, which can kill the end-point singularity [Li, PPNP 51, 85 (2003) and reference therein.]

- The divergent double Log terms $\alpha_s \ln^2[Qb]$ can be summed to all orders, which lead to the Sudakov factor, the long-distance contributions in the end-point region can therefore be suppressed effectively.

  The decay amplitude $\mathcal{A}(B \to M_1 M_2)$ can be written as:

$$\begin{aligned} \mathcal{A} \quad &\sim \quad \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \, \mathrm{Tr}\left[C(t)\Phi_B(x_1,b_1)\right. \\ &\quad \left. \Phi_{M_1}(x_2,b_2)\Phi_{M_2}(x_3,b_3)H(x_i,b_i,t)\cdot S_t(x_i)\cdot e^{-S(t)}\right], \quad (2) \end{aligned}$$

  where $S_t(x_i)$ are obtained from threshold resummation, which smears the end-point singularities on x_i; while $e^{-S(t)}$ is the Sudakov form factor resulting from the overlap of soft and collinear divergences, which suppresses the soft dynamics effectively.

- In pQCD, the form factors can be calculated perturbatively, the only inputs are the wave functions of the mesons involved.

- The pQCD predictions for $\mathcal{A}_{CP}(B \to K^+\pi^-)$ agree well with the measured value.

- For most studied decays, the pQCD predictions for their BR's are consistent with those based on the QCDF, or SCET approach.

- In pQCD, besides the emission diagrams, one can also calculate the annihilation diagrams, which can provides a large strong phase and then results in a large CP violation.

- The $B \to K\eta^{(\prime)}$ decay has been calculated by using the pQCD approach at Leading Order: [Kou and Sanda, PL B525(2002)240; Charng, Kurimoto and Li, PR D74, 074024 (2006)];

- Very recently, a systematic LO pQCD calculations of forty nine $B_s \to PP, PV, VV$ decays have been done. [Ali et al., hep-ph/0703162];

- $B \to K\pi$ and some $B \to PV$ decays have been calculated by employing the pQCD approach at NLO level [Li, Mishima, Sanda, PR D72, 114005(2005), PR D74, 094020 (2006)].

- My group is now preparing for the NLO calculations of B/B_s meson decays in pQCD approach.

⋆ Mixing schemes

- "Singlet-octet" basis: $\theta_8 \approx -21^\circ$, $\theta_1 \approx -9^\circ$;

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_8 & -\sin\theta_1 \\ \sin\theta_8 & \cos\theta_1 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \qquad (3)$$

$$\eta_8 = \frac{1}{\sqrt{6}}\left(u\bar{u} + d\bar{d} - 2s\bar{s}\right), \quad \eta_1 = \frac{1}{\sqrt{3}}\left(u\bar{u} + d\bar{d} + s\bar{s}\right). \qquad (4)$$

- "Quark-flavor" basis: $\eta_q = (u\bar{u} + d\bar{d})/2, \eta_s = s\bar{s}$;

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = U(\phi)\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \qquad (5)$$

where ϕ is the mixing angle with $\phi = 39.3^\circ \pm 1.0^\circ$;

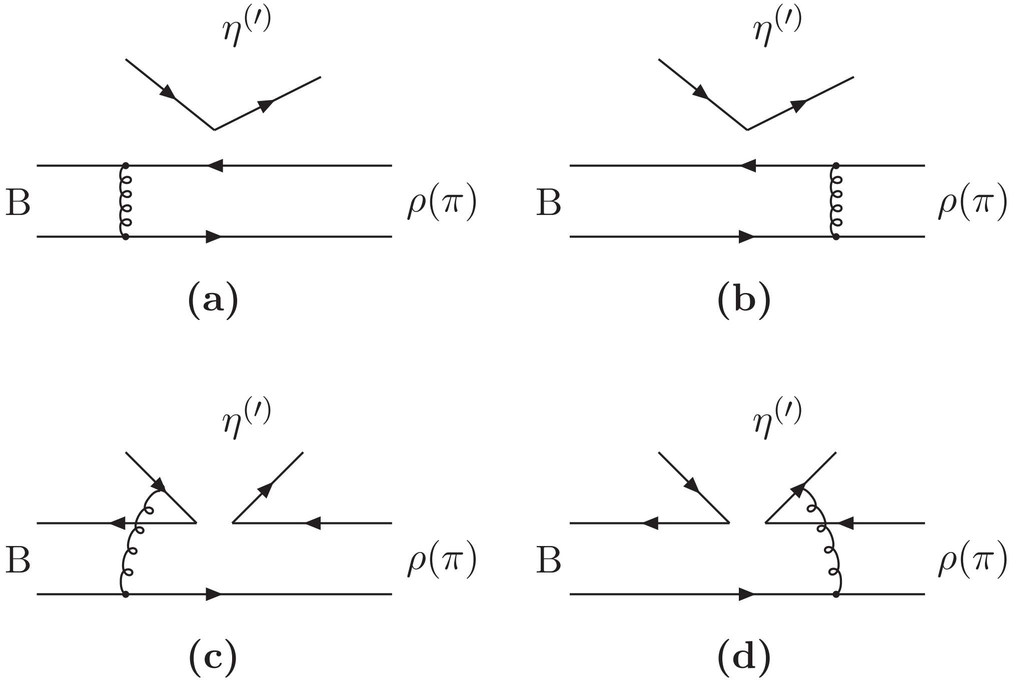


Figure 1: Emission diagrams for $B \to \rho(\pi)\eta^{(\prime)}$ decays. The form factors $F_{0,1}^{B\to\rho}(0)$ and $F_{0,1}^{B\to\pi}(0)$ can be extracted from the factorizable emission diagram (a) and (b).

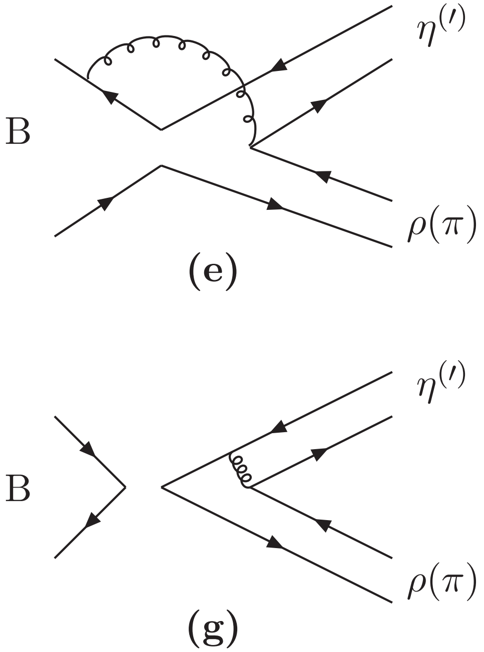


Figure 2: The annihilation diagrams for $B \to \rho(\pi)\eta^{(\prime)}$ decays. Which can contribute a large strong phase.

2. Branching Ratios

We here take the decay $B \to \rho\eta'$ as an example. Both tree and penguin diagrams contribute to this decay:

$$\begin{aligned} \mathcal{M}(B \to \rho\eta') &= V_{ub}^* V_{ud} T - V_{tb}^* V_{td} P = V_{ub}^* V_{ud} T \left[1 + z e^{i(\alpha+\delta)} \right], \quad (6) \\ z &= \left| \frac{V_{tb}^* V_{td}}{V_{ub}^* V_{ud}} \right| \left| \frac{P}{T} \right|, \qquad \alpha = \arg \left[- \frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right], \end{aligned}$$

where α is the usual CKM angle, δ is the difference between the strong phase from the "Tree" and "Penguin"-diagram.

The corresponding CP-conjugate decay amplitude can be written as

$$\overline{\mathcal{M}} = V_{ub}V_{ud}^{*}T - V_{tb}V_{td}^{*}P = V_{ub}V_{ud}^{*}T\left[1 + ze^{i(-\alpha+\delta)}\right] \qquad (7)$$

The CP-averaged branching ratio can be defined as

$$BR(B \to \rho\eta') = \tau_B \frac{G_F^2}{32\pi M_B}\left(1 - \frac{M_V^2}{M_B^2}\right)\left\{|\mathcal{M}|^2 + |\overline{M}|^2\right\} \qquad (8)$$

Table 1: The theoretical predictions for the branching ratios (in units of 10^{-6}) of $B \to \rho\eta^{(\prime)}, \pi\eta^{(\prime)}$ decays in the pQCD, QCDF approach, and date as given by HFAG.

Decays	pQCD	QCDF	HFAG
$B^\pm \to \rho^\pm\eta$	$8.5^{+3.4}_{-2.3}$	$9.4^{+5.9}_{-4.8}$	5.4 ± 1.2
$B^\pm \to \rho^\pm\eta'$	$8.7^{+3.3}_{-2.7}$	$6.3^{+4.0}_{-3.3}$	$9.1^{+3.7}_{-2.8}$
$B^0 \to \rho^0\eta$	$0.02^{+0.10}_{-0.02}$	$0.03^{+0.17}_{-0.10}$	< 1.5
$B^0 \to \rho^0\eta'$	$0.06^{+0.12}_{-0.02}$	$0.01^{+0.12}_{-0.06}$	< 0.13
$B^\pm \to \pi^\pm\eta$	$4.1^{+1.5}_{-1.1}$	$4.7^{+2.7}_{-2.3}$	4.4 ± 0.4
$B^\pm \to \pi^\pm\eta'$	$2.4^{+0.9}_{-0.6}$	$3.1^{+1.9}_{-1.7}$	$2.6^{+0.6}_{-0.5}$
$B^0 \to \pi^0\eta$	0.23 ± 0.08	$0.28^{+0.48}_{-0.28}$	< 1.5
$B^0 \to \pi^0\eta'$	0.19 ± 0.05	$0.17^{+0.33}_{-0.17}$	< 0.13

Table 2: The theoretical predictions for the branching ratios of $B \to (\omega, \phi, \eta^{(\prime)})\eta^{(\prime)}$ decays in the pQCD, QCDF approach, and date as given by HFAG.

Decays	pQCD	QCDF	HFAG
$B^0 \to \omega\eta$	$\left(2.7^{+1.1}_{-1.0}\right) \times 10^{-7}$	$3.1^{+4.6}_{-2.7} \times 10^{-7}$	$< 17 \times 10^{-7}$
$B^0 \to \omega\eta'$	$\left(0.75^{+0.37}_{-0.33}\right) \times 10^{-7}$	$\left(2.0^{+3.4}_{-1.8}\right) \times 10^{-7}$	$< 28 \times 10^{-7}$
$B^0 \to \phi\eta$	$\left(6.3^{+3.3}_{-1.9}\right) \times 10^{-9}$	1×10^{-9}	$< 6 \times 10^{-7}$
$B^0 \to \phi\eta'$	$\left(7.5^{+3.5}_{-2.6}\right) \times 10^{-9}$	1×10^{-9}	$< 10 \times 10^{-7}$
$B^0 \to \eta\eta$	$\left(0.67^{+0.32}_{-0.25}\right) \times 10^{-7}$	$\left(1.6^{+4.5}_{-1.9}\right) \times 10^{-7}$	$< 1.8 \times 10^{-6}$
$B^0 \to \eta\eta'$	$(0.18 \pm 0.11) \times 10^{-7}$	$\left(1.6^{+6.1}_{-1.8}\right) \times 10^{-7}$	$< 1.7 \times 10^{-6}$
$B^0 \to \eta'\eta'$	$\left(0.11^{+0.12}_{-0.09}\right) \times 10^{-7}$	$\left(0.6^{+2.5}_{-0.7}\right) \times 10^{-7}$	$< 2.4 \times 10^{-6}$

Table 3: The theoretical predictions for the branching ratios of $B_s \to (\pi, \rho, \omega, \eta^{(\prime)})\eta^{(\prime)}$ decays in the pQCD, QCDF approach, and date as given by HFAG.

Decays	pQCD	QCDF	HFAG
$B_s^0 \to \pi^0\eta$	$\left(0.86^{+1.12}_{-0.33}\right) \times 10^{-7}$	$0.75^{+0.35}_{-0.30} \times 10^{-7}$	$< 1.0 \times 10^{-3}$
$B_s^0 \to \pi^0\eta'$	$\left(1.86^{+1.76}_{-0.69}\right) \times 10^{-7}$	$(1.1 \pm 0.24) \times 10^{-7}$	$< 1.0 \times 10^{-3}$
$B_s^0 \to \rho^0\eta$	$\left(0.7^{+0.6}_{-0.4}\right) \times 10^{-7}$	$1.7^{+0.8}_{-0.7} \times 10^{-7}$	$--$
$B_s^0 \to \rho^0\eta'$	$\left(1.0^{+0.8}_{-0.5}\right) \times 10^{-7}$	$\left(2.5^{+1.2}_{-1.0}\right) \times 10^{-7}$	$--$
$B_s^0 \to \omega\eta$	$\left(0.20^{+0.15}_{-0.03}\right) \times 10^{-7}$	$0.12^{+0.39}_{-0.10} \times 10^{-7}$	$--$
$B_s^0 \to \omega\eta'$	$\left(1.3^{+0.5}_{-0.4}\right) \times 10^{-7}$	$\left(0.24^{+0.93}_{-0.21}\right) \times 10^{-7}$	$--$
$B_s^0 \to \eta\eta$	$\left(14.2^{+18.0}_{-7.5}\right) \times 10^{-6}$	$\left(15.6^{+17.0}_{-9.2}\right) \times 10^{-6}$	$< 1.8 \times 10^{-3}$
$B_s^0 \to \eta\eta'$	$\left(12.4^{+18.2}_{-7.0}\right) \times 10^{-6}$	$\left(54.0^{+52.8}_{-29.1}\right) \times 10^{-6}$	$--$
$B_s^0 \to \eta'\eta'$	$\left(9.2^{+15.3}_{-4.9}\right) \times 10^{-6}$	$\left(41.7^{+47.7}_{-24.9}\right) \times 10^{-6}$	$--$

Table 4: The theoretical predictions for the branching ratios (in units of 10^{-6}) of $B_s \to \phi\eta^{(\prime)}$ decays in the pQCD, QCDF approach.

Decays	pQCD	pQCD	QCDF	HFAG
$B^0 \to \phi\eta$	$26.6^{+18.2}_{-10.8}$	1.8 ± 0.6	$0.12^{+1.14}_{-0.23}$	$--$
$B^0 \to \phi\eta'$	$20.0^{+16.3}_{-9.1}$	$3.6^{+1.3}_{-1.0}$	$0.05^{+1.18}_{-0.19}$	$--$

$\star$ Some remarks on the branching ratios:

- For all considered B^0 decays, the TH predictions for the BR's in both the pQCD and QCDF approach are consistent with each other, and agree well with the data.

- For most considered B_s decays, the theoretical predictions for the BR's in both the pQCD and QCDF approach are consistent with each other, since the TH uncertainties are still large, and the data are still poor.

- For $B_s \to \phi\eta$ and $\phi\eta'$ decays, however, there is a clear difference between the theoretical predictions. We are now checking our calculations.

- Following Li et al., we also estimated the gluonic contributions to the decays under study and found that such contributions are small: less than 10% in general.

2. CP-violating Asymmetries

♣ The direct CP violation for $B^{\pm}$ meson decays:

$$\mathcal{A}_{CP}^{dir} = \frac{|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2}{|\overline{\mathcal{M}}|^2 + |\mathcal{M}|^2} = \frac{2z \sin\alpha \sin\delta}{1 + 2z\cos\alpha\cos\delta + z^2}. \tag{9}$$

♣ $B^0 - \overline{B}^0$ mixing and time-dependent CP violating asymmetries:

$$\begin{aligned} A_{CP} &= A_{CP}^{dir}\cos(\Delta m \Delta t) + A_{CP}^{mix}\sin(\Delta m\Delta t), \\ A_{CP}^{dir} &= \frac{\left|\lambda_{CP}\right|^2 - 1}{1 + |\lambda_{CP}|^2}, \qquad A_{CP}^{mix} = \frac{2Im(\lambda_{CP})}{1+|\lambda_{CP}|^2}, \end{aligned} \tag{10}$$

where for $b \to d$ transition,

$$\lambda_{CP} = \frac{V_{tb}^* V_{td} \langle \bar{f} | H_{eff} | \overline{B}^0 \rangle}{V_{tb} V_{td}^* \langle f | H_{eff} | B^0 \rangle} = e^{2i\alpha} \frac{1 + z e^{i(\delta-\alpha)}}{1 + z e^{i(\delta+\alpha)}}. \qquad (11)$$

- ♣ For B_s decays, a non-zero ratio $(\Delta\Gamma/\Gamma)_{B_s}$ is expected in the SM: For $B \to \eta\eta$ decay, for example, three parameters are defined as

$$\mathcal{A}_{CP}^{dir} = \frac{|\lambda_{CP}|^2 - 1}{1 + |\lambda|^2}, \quad \mathcal{A}_{CP}^{mix} = \frac{2Im(\lambda)}{1 + |\lambda|^2}, \quad \mathcal{A}_{\Delta\Gamma_s} = \frac{2Re(\lambda)}{1 + |\lambda|^2}, \qquad (12)$$

$$\lambda = \eta_f \frac{V_{tb}^* V_{ts} \langle f | H_{eff} | \bar{B}_s^0 \rangle}{V_{tb} V_{ts}^* \langle f | H_{eff} | B_s^0 \rangle} \approx \frac{\langle f | H_{eff} | \bar{B}_s^0 \rangle}{\langle f | H_{eff} | B_s^0 \rangle}, \qquad (13)$$

in a very good approximation. The term $\mathcal{A}_{\Delta\Gamma_s}$ is related to the presence of a non-negligible $\Delta\Gamma_s$.

Table 5: The CP violating asymmetries of $B \to \rho\eta^{(\prime)}, \pi\eta^{(\prime)}$ decays in the pQCD, QCDF approach. The data are: $\mathcal{A}_{CP}^{dir}(\rho^{\pm}\eta)^{exp} = 0.03 \pm 0.16$, $\mathcal{A}_{CP}^{dir}(\pi^{\pm}\eta)^{exp} = -0.11 \pm 0.08$, $\mathcal{A}_{CP}^{dir}(\pi^{\pm}\eta')^{exp} = 0.14 \pm 0.15$.

Decays	pQCD		QCDF	
	$\mathcal{A}_{CP}^{dir}$	$\mathcal{A}_{CP}^{mix}$	$\mathcal{A}_{CP}^{dir}$	$\mathcal{A}_{CP}^{mix}$
$B^{\pm} \to \rho^{\pm}\eta$	-13%	$--$	-2%	$--$
$B^{\pm} \to \rho^{\pm}\eta'$	-18%	$--$	$+4\%$	$--$
$B^{0} \to \rho^{0}\eta$	-41%	$+25\%$	16%	$--$
$B^{0} \to \rho^{0}\eta'$	-27%	$+11\%$	$+21\%$	$--$
$B^{\pm} \to \pi^{\pm}\eta$	-37%	$--$	-15%	$--$
$B^{\pm} \to \pi^{\pm}\eta'$	-33%	$--$	-9%	$--$
$B^{0} \to \pi^{0}\eta$	-37%	$+67\%$	-18%	$--$
$B^{0} \to \pi^{0}\eta'$	-33%	$+67\%$	-19%	$--$

Table 6: The CP violating asymmetries of $B \to (\omega, \phi, \eta^{(\prime)})\eta^{(\prime)}$ decays in the pQCD, QCDF approach.

Decays	pQCD		QCDF	
	$\mathcal{A}_{CP}^{dir}$	$\mathcal{A}_{CP}^{mix}$	$\mathcal{A}_{CP}^{dir}$	$\mathcal{A}_{CP}^{mix}$
$B^0 \to \omega\eta$	-69%	67%	-33%	$--$
$B^0 \to \omega\eta'$	$+14\%$	$+66\%$	$+1\%$	$--$
$B^0 \to \phi\eta$	0	0	$--$	$--$
$B^0 \to \phi\eta'$	0	0	$--$	$--$
$B^\pm \to \eta\eta$	$+14\%$	$+91\%$	$+63\%$	$--$
$B^\pm \to \eta\eta'$	$+76\%$	$+6\%$	$+56\%$	$--$
$B^0 \to \eta'\eta'$	$+86\%$	$+50\%$	$+46\%$	$--$

♣ For $B \to \phi\eta^{(\prime)}$ decays, there is no CPV, since they involve only the penguin contributions and one type of CKM element $\xi_t = V_{tb}^* V_{td}$.

Table 7: The CPV asymmetries of $B_s \to M\eta^{(\prime)}$ decays.

Decays	pQCD		QCDF	
	$\mathcal{A}_{CP}^{dir}$	$\mathcal{A}_{CP}^{mix}$	$\mathcal{A}_{CP}^{dir}$	$\mathcal{A}_{CP}^{mix}$
$B_s^0 \to \pi^0\eta$	-5%	-2%	$--$	$--$
$B_s^0 \to \pi^0\eta'$	-9%	$+22\%$	$+28\%$	$--$
$B_s^0 \to \rho^0\eta$	$+5\%$	-4%	$+28\%$	$--$
$B_s^0 \to \rho^0\eta'$	-25%	$+19\%$	$+29\%$	$--$
$B_s^0 \to \omega\eta$	$+8\%$	$+8\%$	$+1\%$	$--$
$B_s^0 \to \omega\eta'$	-8%	$+25\%$	$+4\%$	$--$
$B_s^0 \to \phi\eta$	0.5%	-0.5%	$+23\%$	$--$
$B_s^0 \to \phi\eta'$	-0.5%	0.5%	-58%	$--$
$B_s^0 \to \eta\eta$	-0.2%	-0.3%	-2%	$--$
$B_s^0 \to \eta\eta'$	0.6%	-0.8%	$+0.4\%$	$--$
$B_s^0 \to \eta'\eta'$	-0.8%	$+2\%$	$+2\%$	$--$

3. Sources of Uncertainties

- Higher order corrections. Systematic NLO calculations are clearly needed.
- The DAs of light mesons $K, \pi, \eta^{(\prime)}, \rho, \cdots$;
- The factorization scale t, the chiral mass $m_0^{\pi}, m_0^{K}, \cdots$.
- Heavy meson DAs, and meson decay constants.
- Mixing schemes of $\eta - \eta'$ system: "singlet-octet" or "quark-flavor".
- CKM elements, light quark masses!

4. Summary and Discussions

- For $B/B_s \to M\eta^{(\prime)}$ decays, the pQCD predictions for BRs agree well with the QCDF predictions and with the available data, except for $B_s \to \phi\eta^{(\prime)}$ decays.

- The pQCD predictions of the CP-violating asymmetries are generally larger than the QCDF ones. For B_s decays, however, they are small. The current date are still poor or absent now.

- The contribution of possible gluonic component of $\eta^{(\prime)}$ are small in size: less than 10% for most considered decays. Of course, more works about the various gluonic contributions to $B \to M\eta'$ decays need to be done.

- The theoretical uncertainties are still large, NLO calculations needed.

Thanks For Your Attention

Backup slides

♣ One kind of gluonic contribution to $F_{0,1}^{B\to\eta'}$:

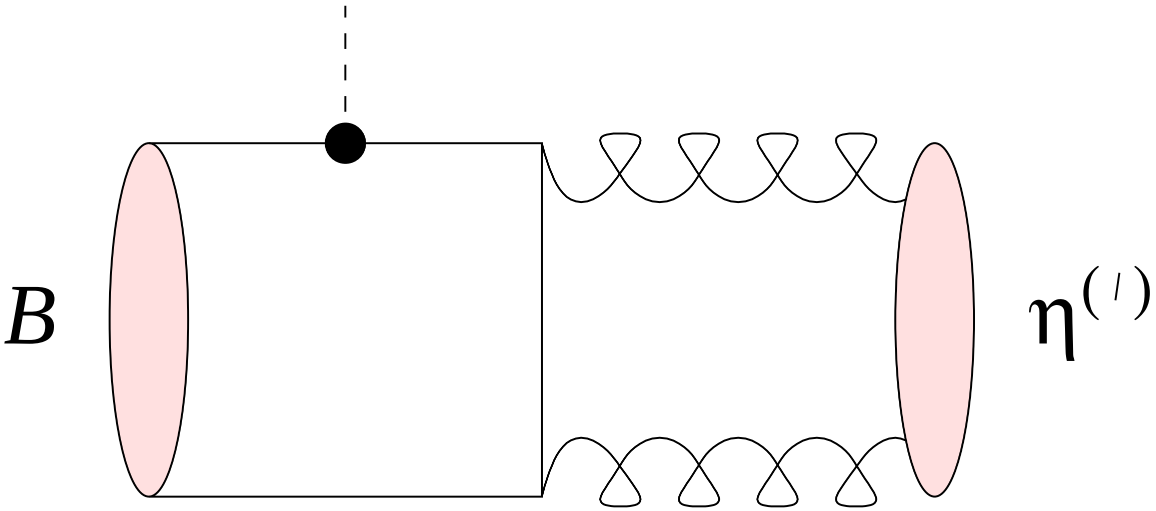
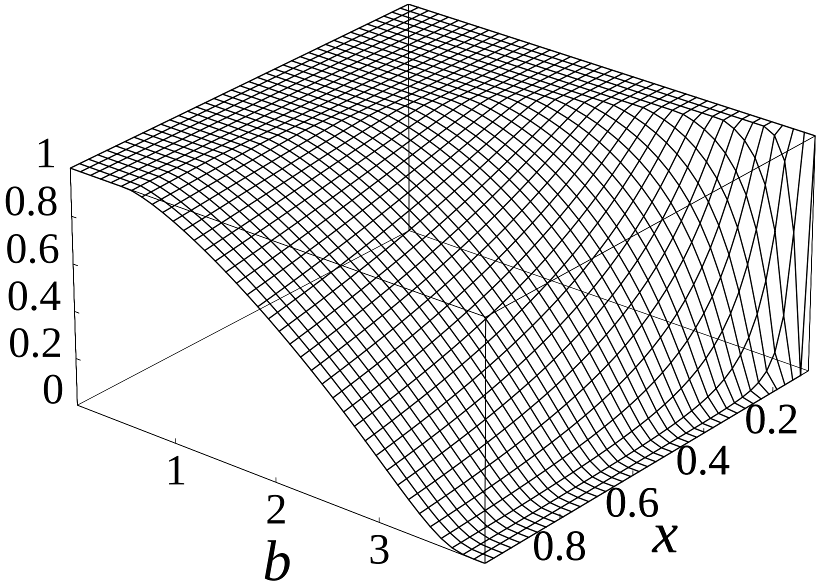


Figure 3: Gluonic contribution to the $B \to \eta^{(\prime)}$ form factors. Another diagram with the two gluons crossed is suppressed. Quoted from Li's paper.

♣ Suppression of Sudakov factor:



♣ In the region of large b, $b \sim b_{max} = 1/\Lambda_{QCD}$, Sudakov factor is very small, close to zero. The long-distance (large b) contributions are therefore suppressed effectively.